Lecture-3

Magnetic forces, materials and devices: Forces due to magnetic field, magnetic torque and moment, a magnetic dipole, magnetization in materials, magnetic boundary conditions, inductors and inductances, magnetic energy.

Introduction

- Question: Why do some materials respond to magnetic fields, while others do not?

- Answer: Magnetic materials have a property known as Magnetization as quantified in the relative permeability constant (μ_r).

Description of Magnetic Material Properties

- For accurate quantitative prediction:

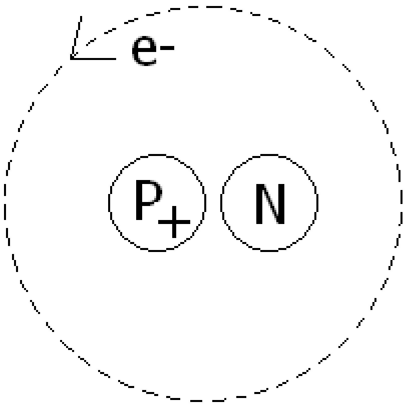
  Quantum Theory is required

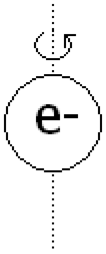
- For qualitative description:

  Orbital Mechanics Model suffices

Orbital Mechanics Model

- Atom has electrons that orbit around its nucleus making a miniature current loop that results in an orbital magnetic moment
- Electron spins around its own axis to produce a significant spin magnetic moment





Orbital Mechanics Model

- The relative contribution of the magnetic moments of each atom and the molecular makeup of a material classifies it as
  - Diamagnetic
  - Paramagnetic
  - Ferromagnetic
  - Antiferromagnetic
  - Ferrimagnetic
  - Superparamagnetic
- Any atom with a magnetic moment in the presence of an applied magnetic field will experience a torque that tends to align it

Domain

- Definition:

  a <u>region</u> within a ferromagnetic material having a <u>large dipole moment</u> due to collections of associated atoms with uncompensated spin moments

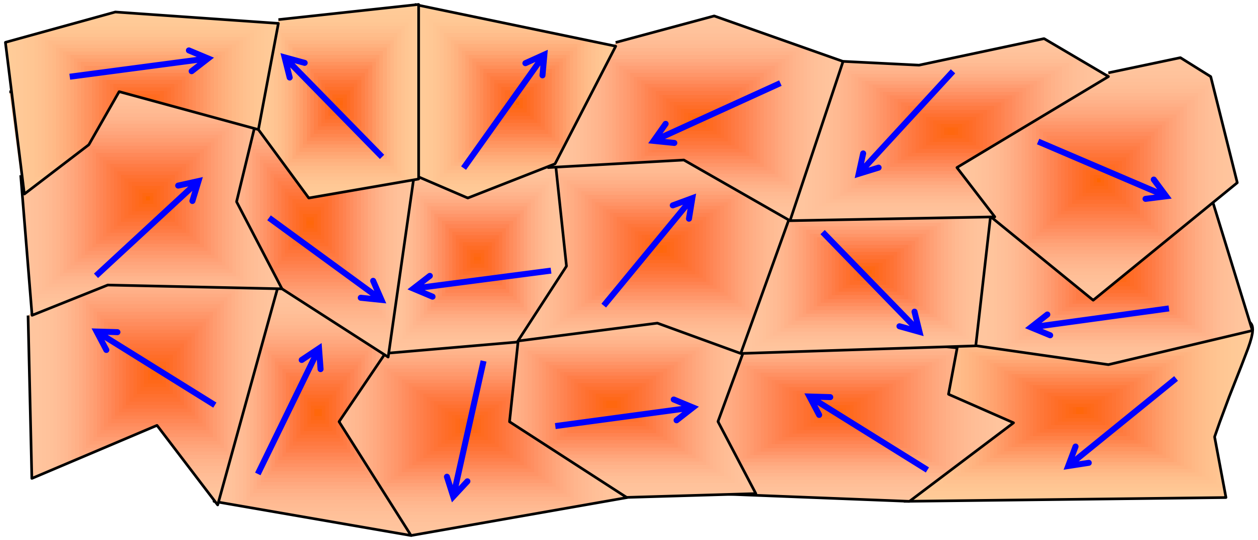
- Shape, size & direction of moment:

  varies between neighboring regions within a <u>crude</u> sample that cancels the effect overall

See also magnetic dipole moments and domains of ferromagnetic materials as illustrated in Fig. 3.36 WW pp. 139, 140.

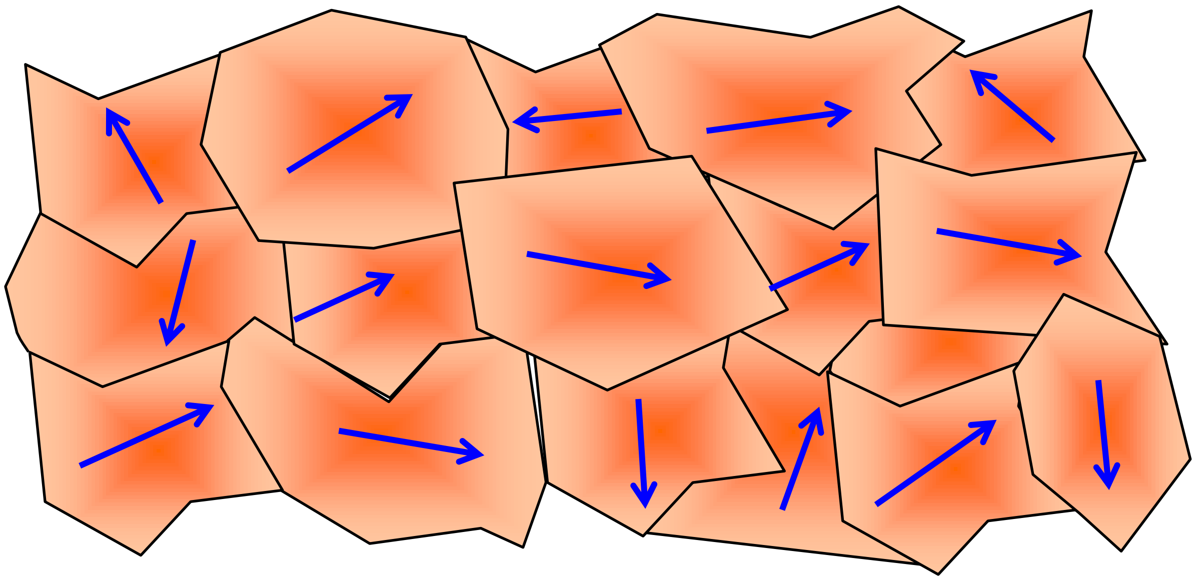
How do we understand ferromagnetism?

Domains: Small regions that have aligned dipole moments are called domains. In unmagnetized iron, the domains are randomly oriented.



How do we understand ferromagnetism?

Domains: In a permanent magnet, the domains tend to be aligned in a particular direction.



Alignment of Magnetic Domains

- Alignment:

  may be achieved by an applied magnetic field

- Upon removal of the external magnetic field,

  domains do not all return to their original state and thus exhibit a magnetic history known as <u>hysteresis</u> (an interesting & practical effect unique to ferromagnetic materials)

See also nonlinear Magnetization curve in Fig. 3.37 WW p. 141

- Examples: Ferromagnetic elements & compounds

  Fe, Ni, Co, BiMn, CuMnSn, etc.

Other Examples of Magnetic Materials Each Class

Class of Magnetic Material	Ex's: Elements & Compounds
Diamagnetic	Bi, H, He, NaCl, Au, Cu, Si, Ge, etc.
Paramagnetic	K, O, etc.
Antiferromagnetic	MnO, NiO, FeS, $CoCl_2$, etc.
Ferrimagnetic	Fe_3O_4 (iron oxide magnetite), $NiFe_2O_4$ (nickel ferrite), etc.
Superparamagnetic	Ferromagnetic particles in a nonferromagnetic matrix

Qualitative Summary of Magnetic Material Properties

Class	m_{orb} vs. m_{spin}	B_{int} vs. B_{appl}	Comments
Dia-	$m_{orb} = -m_{spin}$	$<$	weak effect
Para-	$m_{orb} + m_{spin}$ small	$>$	weak effect
Antiferro-	$<<$	$\cong$	int. canc.
Ferro-	$<<$	$>>$	Domains!
Ferri-	$<<$	$>$	High resist.
Superpara-	$<<$	$>$	Matrix

Applications of Magnetic Materials

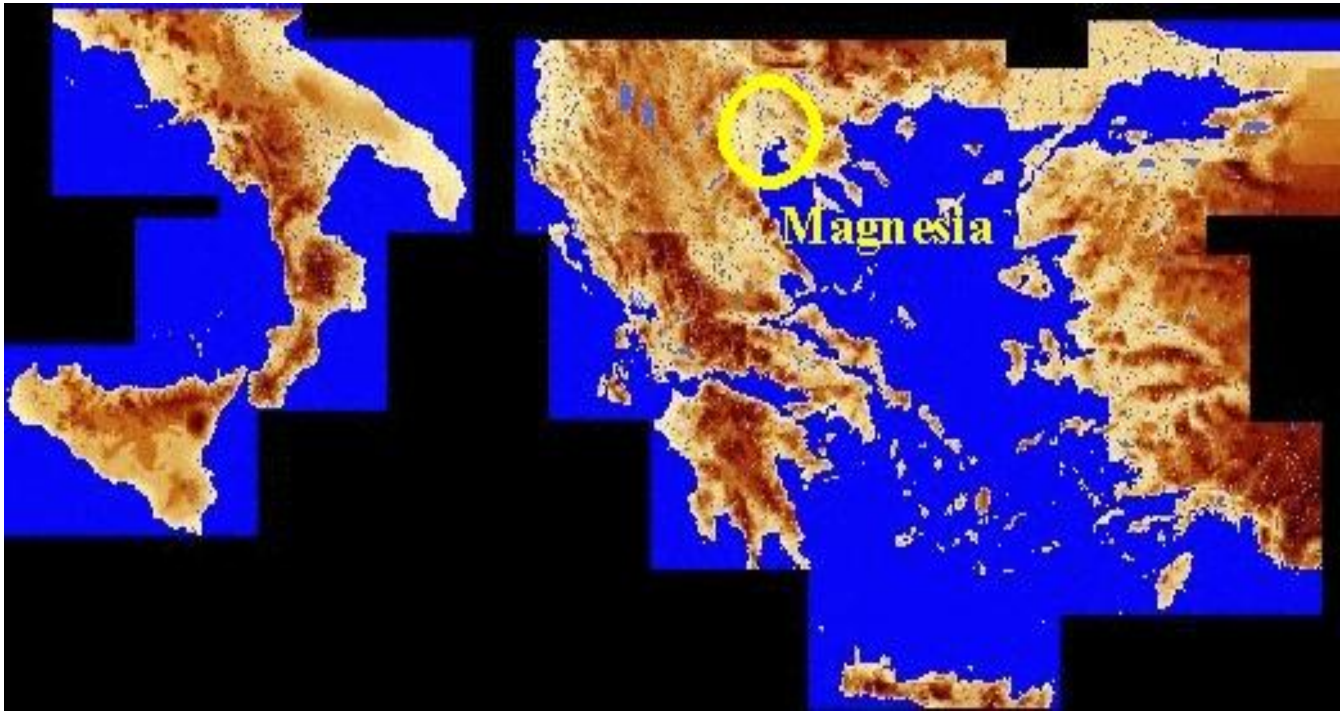
- Ferromagnetic Materials: Permanent Magnets, Magnetic Data Storage, etc.

- Ferrimagnetic Materials: Ferrites commonly used for transformer and/or toroid cores due to their higher resistance that reduces eddy currents that cause ohmic loss

- Superparamagnetic: used to create recording tape for audio or video application

Permanent Magnets

"Magnetism" comes from the region called Magnesia, where loadstone (magnetite) was found.

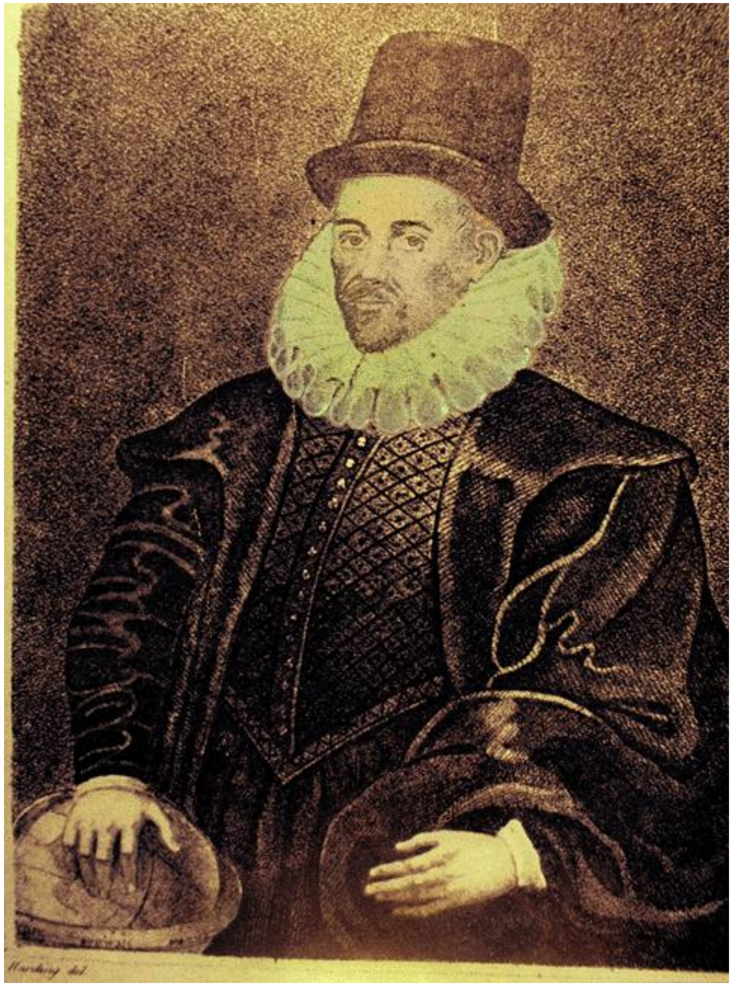


Permanent Magnets

1) Magnetite or loadstone was known from antiquity.
2) Loadstone floating on wood rotates so one end always points north. This is the north pole.
3) If two magnets are placed near other, like poles attract and unlike poles repel.

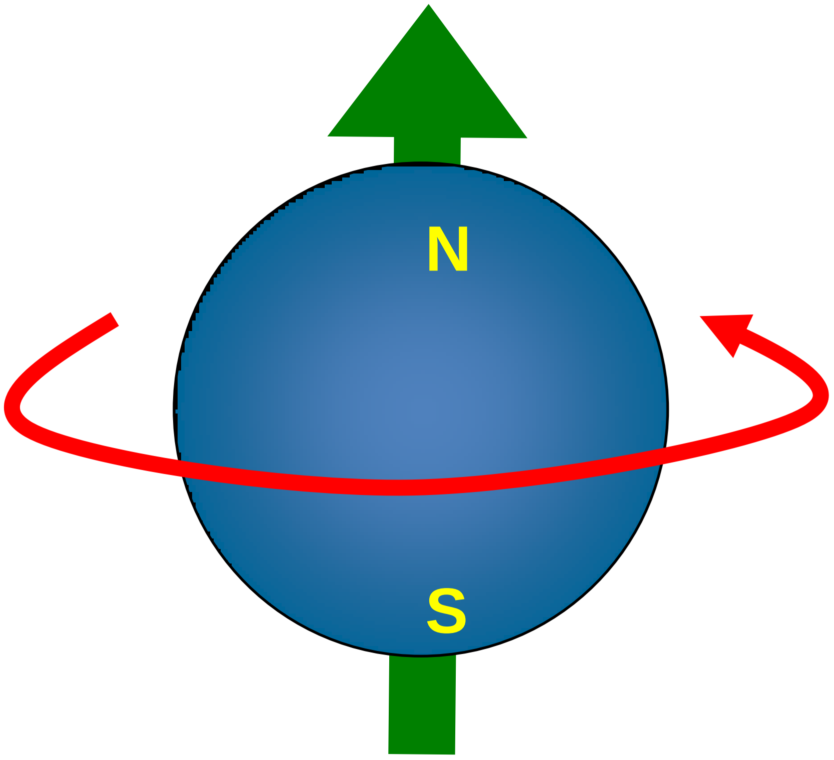
Permanent Magnets

William Gilbert in 1600 published *De Magnete* – where he described magnetism as the “soul of the earth.”

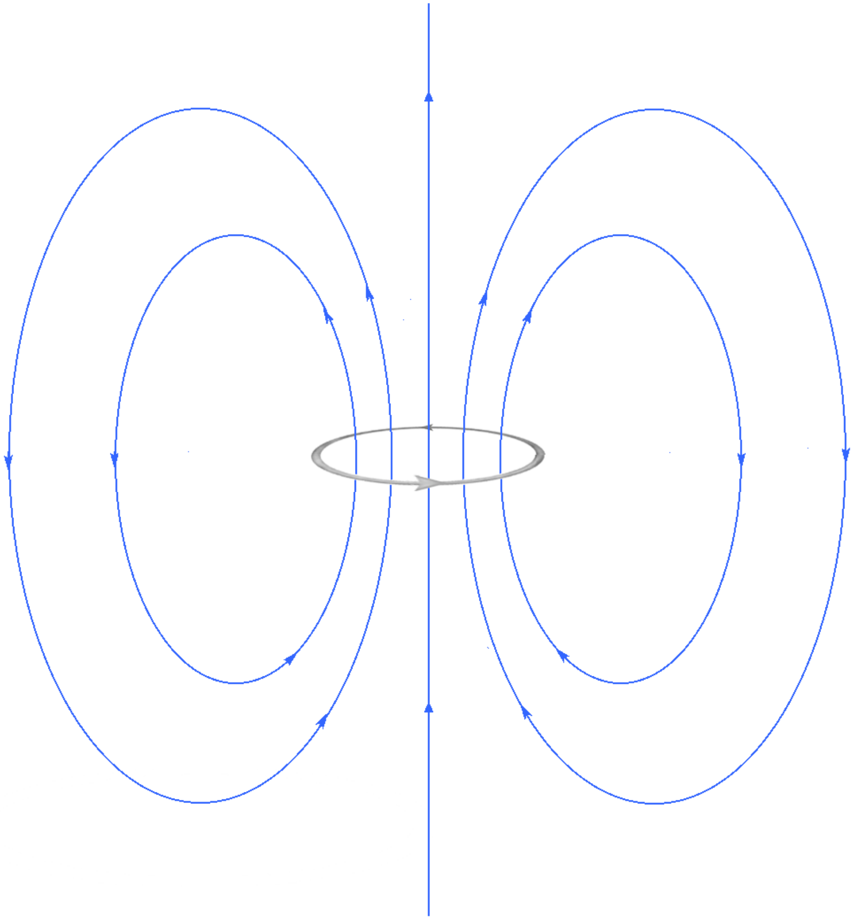


Permanent Magnets

Gilbert: A perfectly spherical magnet spins without stopping – because the earth is a perfect sphere and it's a magnet and it spins without stopping.

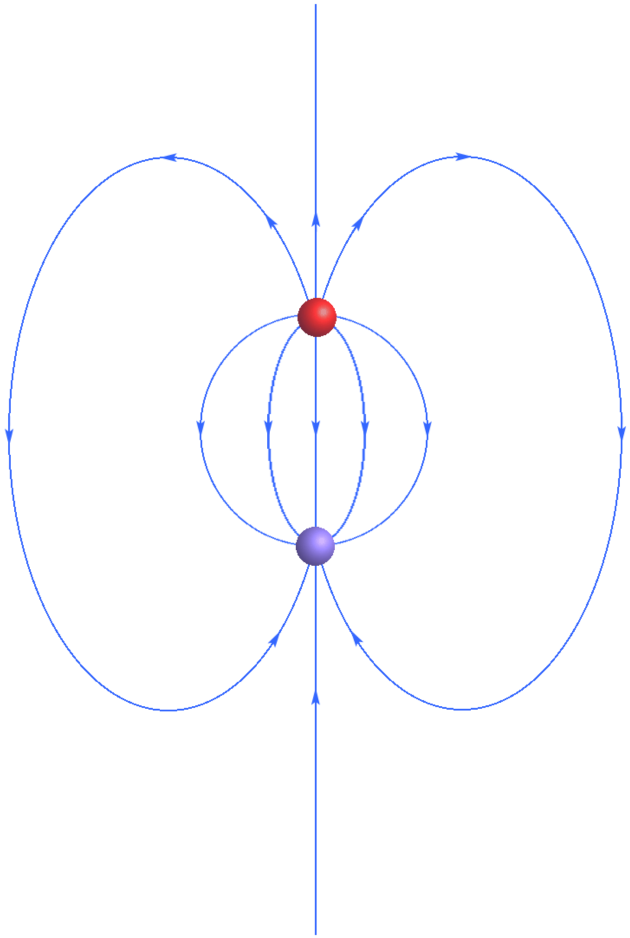


Magnetic fields are generated by movement of electric charges



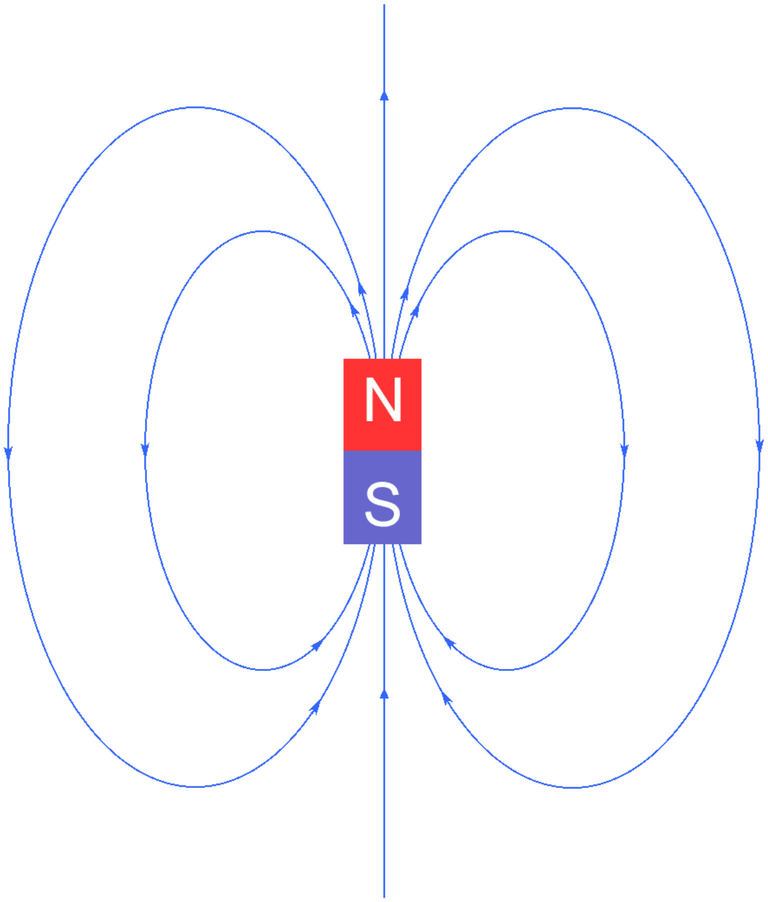
A loop of electric current generates a magnetic dipole field

A magnetic dipole



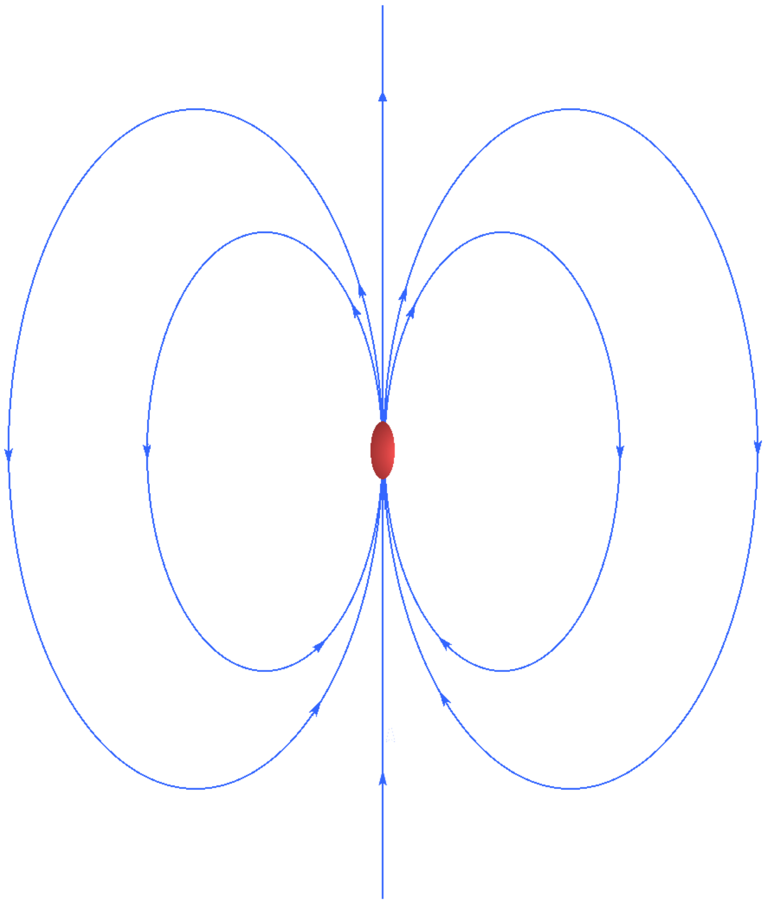
- Field lines run from the North pole to the South pole
- Field lines indicate the direction of force that would be experienced by a North magnetic monopole

A bar magnet



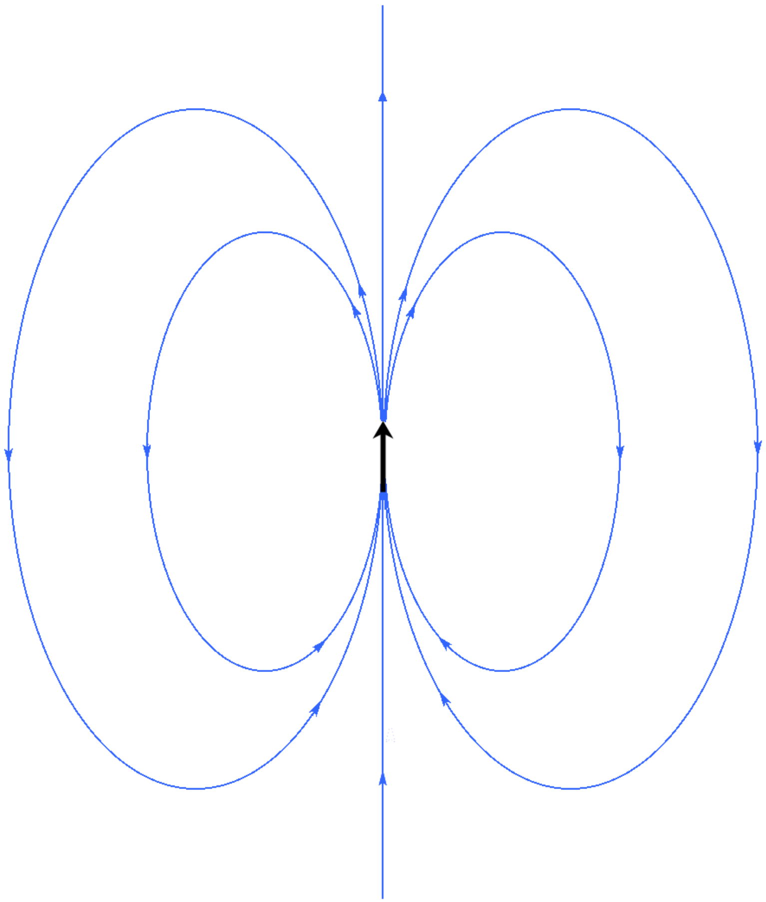
A simple bar magnet behaves like a magnetic dipole

Far field picture



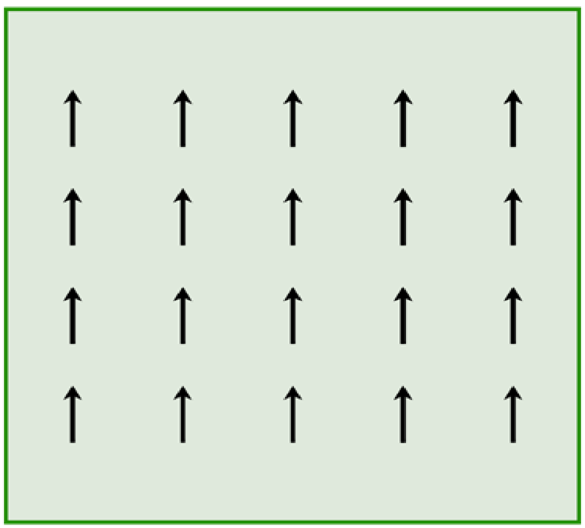
- Sometimes the dipoles are very small compared with their spatial field of influence
- An electron, for example

Schematic representation



- A magnetic dipole is often represented schematically as an arrow.
- The head of the arrow is the North pole.

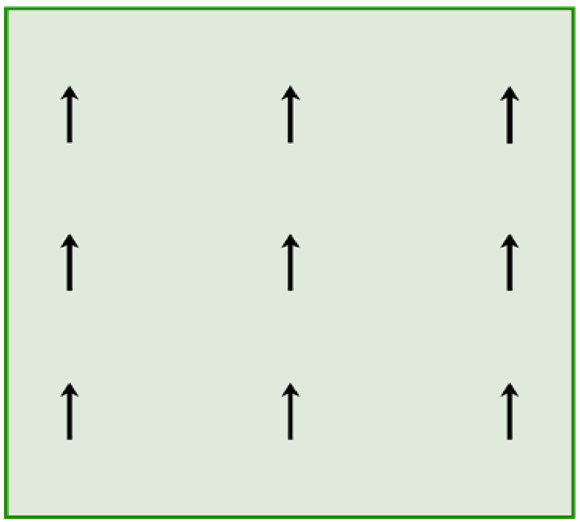
Magnetization, *M*



- Material with a net magnetic moment is magnetized

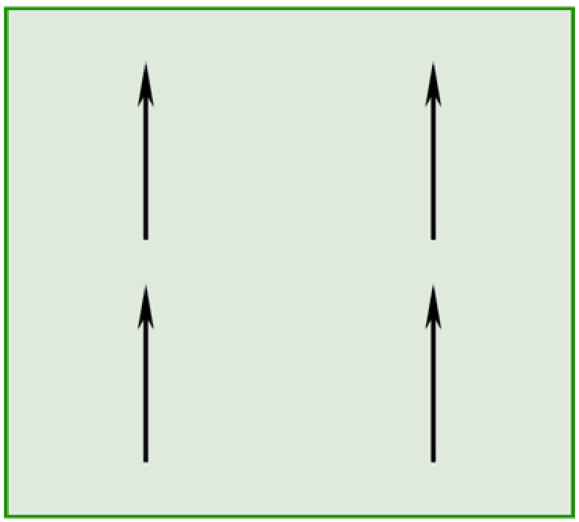
- Magnetization is the magnetic moment per unit volume within the material

Magnetization depends on……..



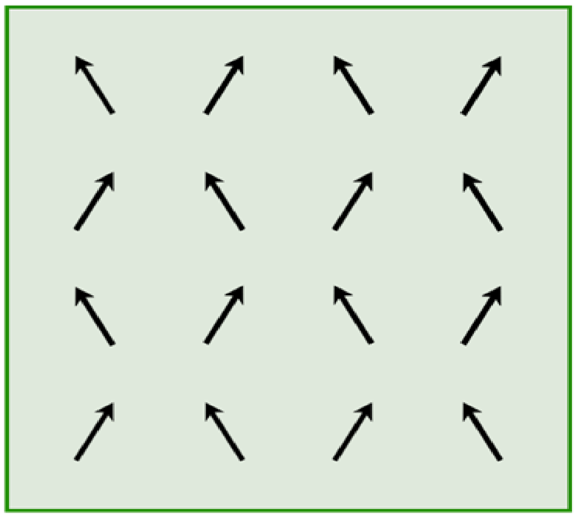
- Number density of magnetic dipole moments within material

Magnetization depends on……..



- Magnitude of the magnetic dipole moments within the material

Magnetization depends on……..

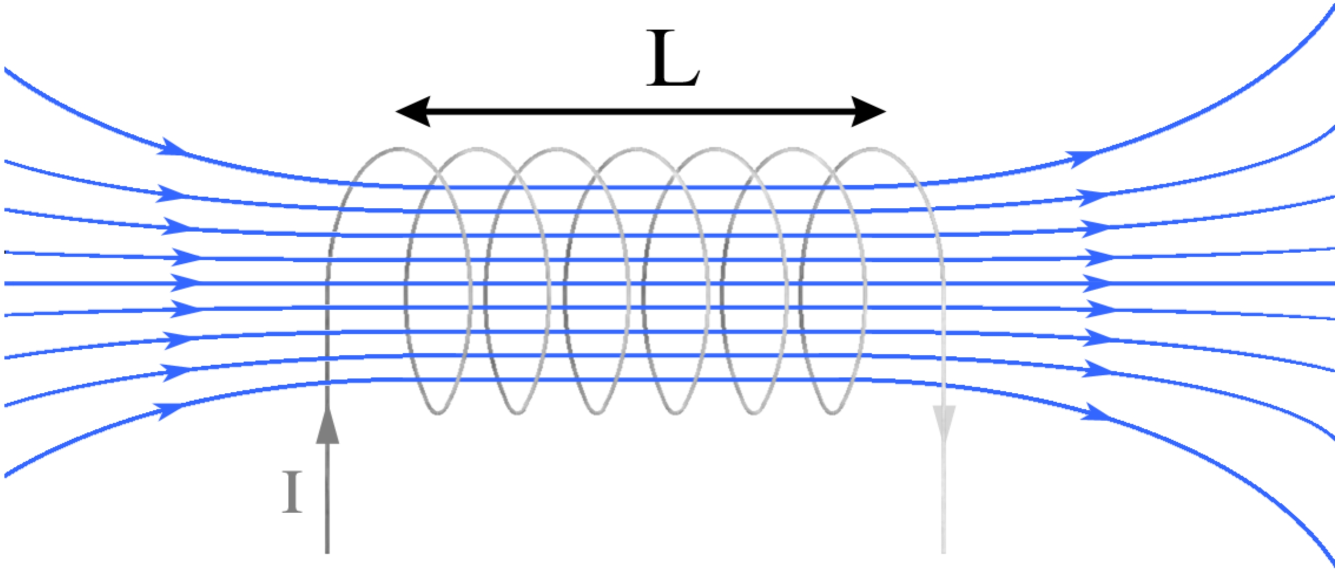


- The arrangement of the magnetic dipoles within the material

Magnetization in materials arises from…….

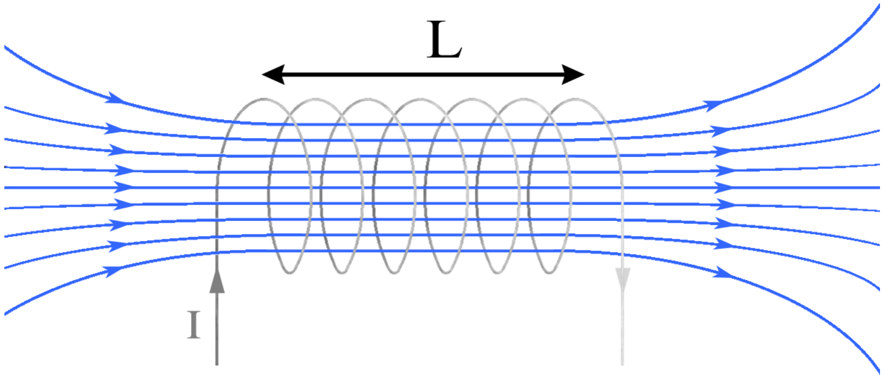
- unpaired electron spins mainly
- the orbital motion of electrons within the material to a lesser extent

Generating a uniform magnetic field in the laboratory



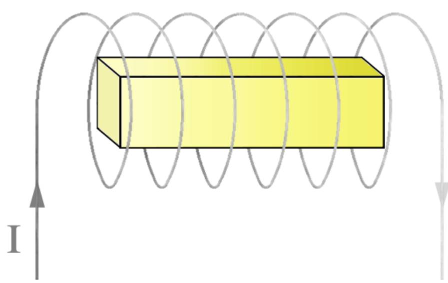
- An electric current run through a conducting coil (solenoid) generates a uniform flux density within the coil

Flux density in vacuum (or air) within coil……..



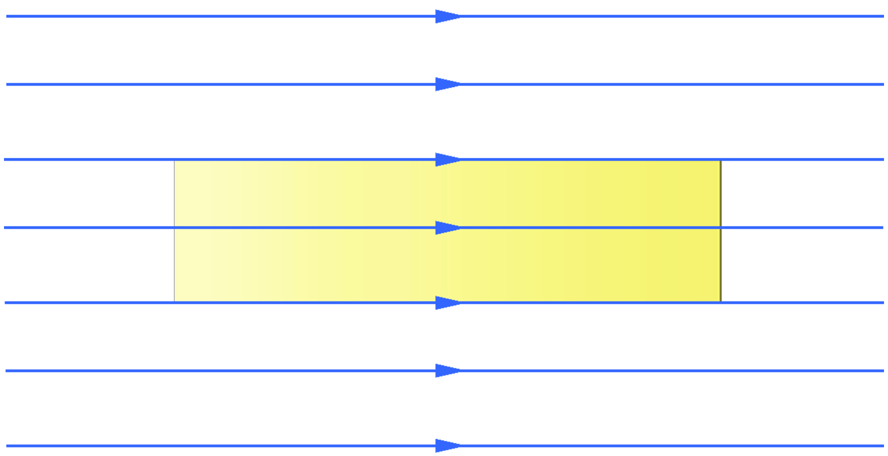
- Increases in proportion to the electric current
- Increases in proportion to the number of turns per unit length in the coil

Inserting a specimen into the coil



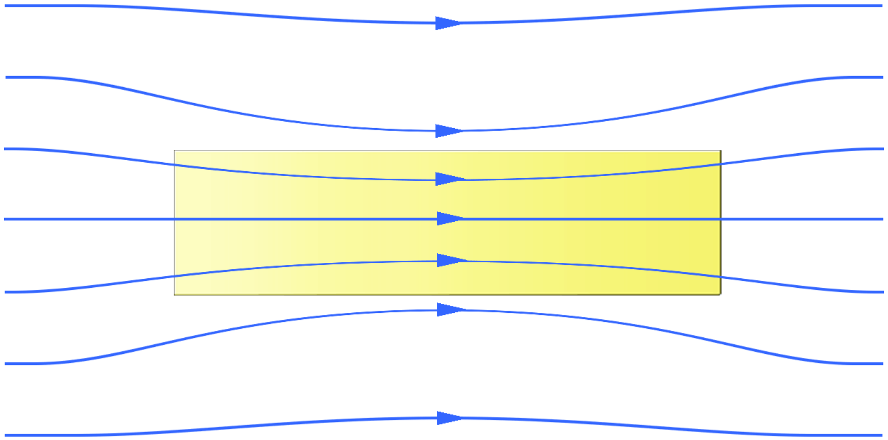
- Generally, the orbital and spin magnetic moments within atoms respond to an applied magnetic field
- Flux lines are perturbed by specimen

Specimen in magnetic field



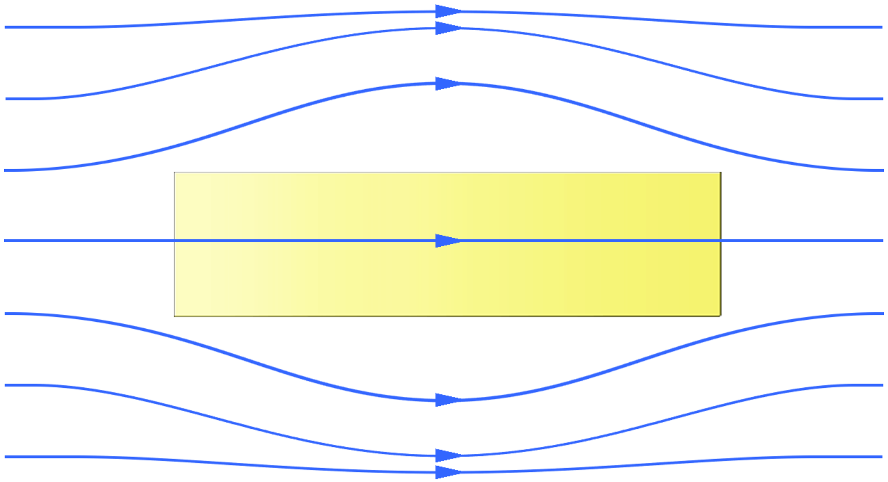
- If specimen has no magnetic response, flux lines are not perturbed

“Magnetic” materials



- “magnetic” materials tend to concentrate flux lines
- Examples: materials containing high concentrations of magnetic atoms such as iron, cobalt

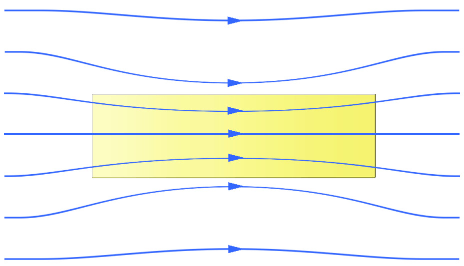
Diamagnetic materials

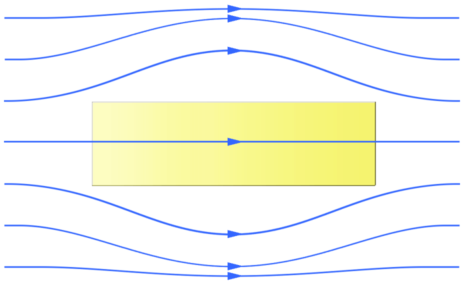


- Diamagnetic materials tend to repel flux lines weakly
- Examples: water, protein, fat

Flux density *B* within material determined by both……

- Geometry and current in solenoid
- Magnetic properties of the material
- Geometry of material



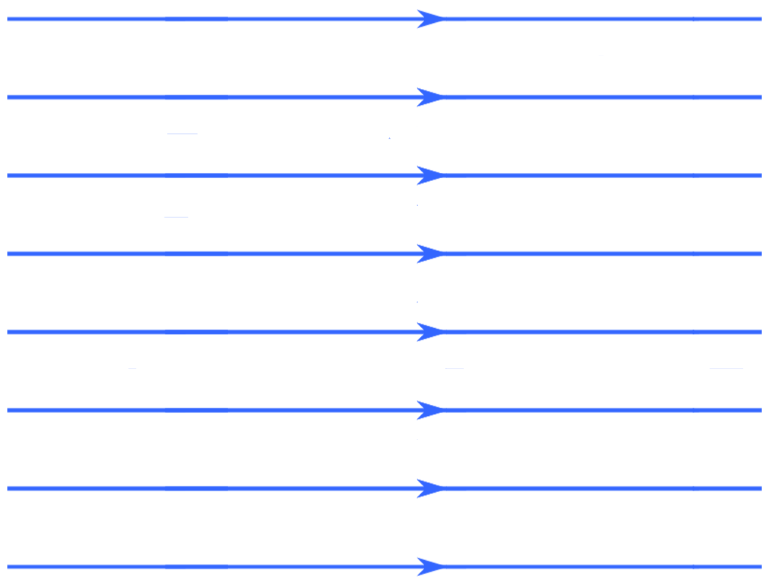


$$\boldsymbol{B} = \mu_0 (\boldsymbol{H} + \boldsymbol{M})$$

The ***H*** Field

- *H* is called the magnetic field strength
- μ_0 is a constant called the permeability of free space

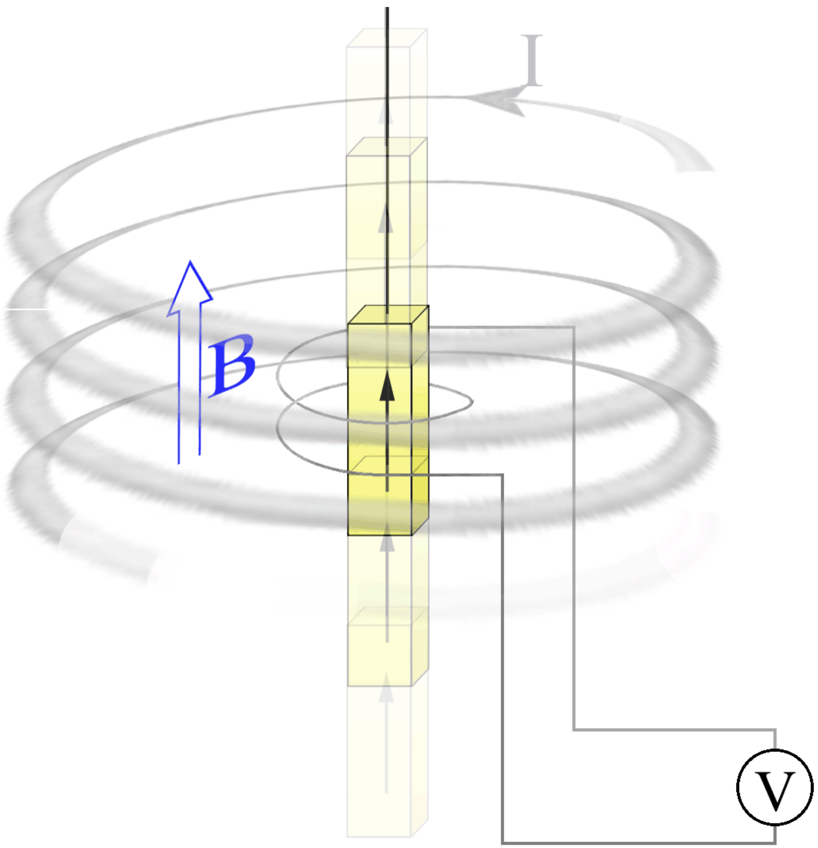
In the absence of material in the solenoid……



- There is no magnetization *M*
- So…..

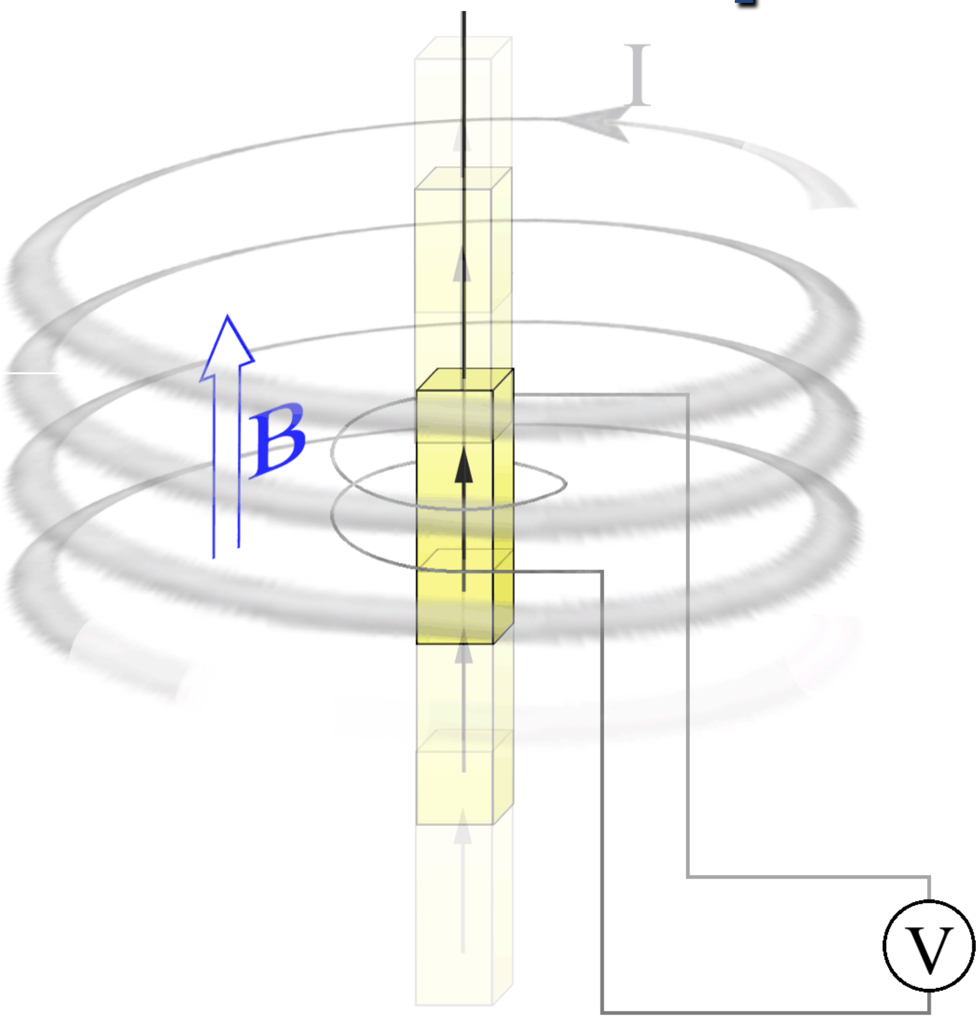
$$\boldsymbol{B} = \mu_0 \boldsymbol{H}$$

Measuring magnetic moment of specimen



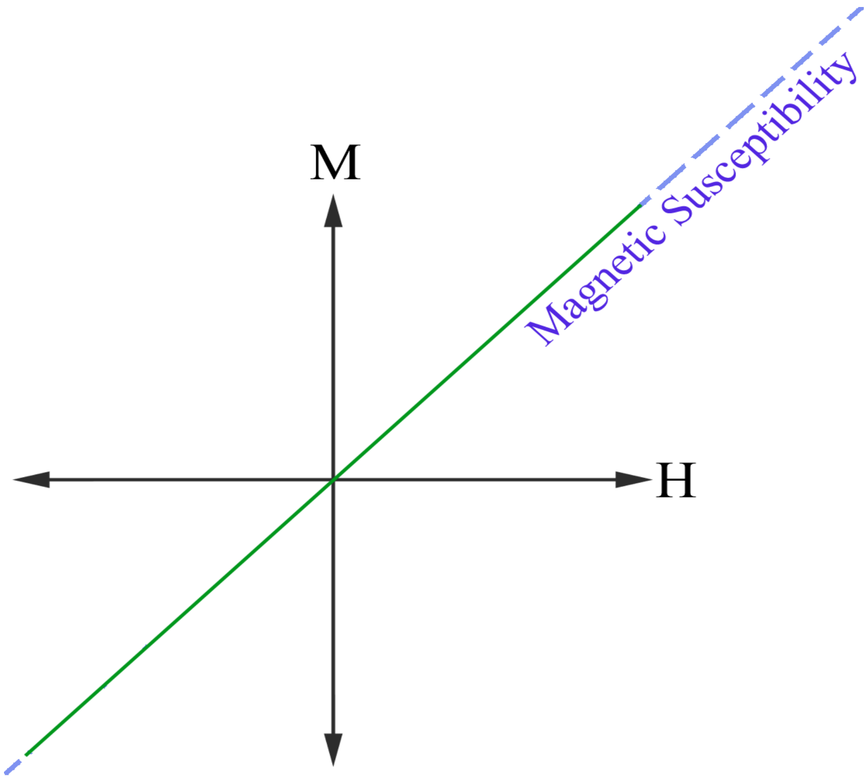
- Pass specimen thru small “sensing” coil
- Measure voltage generated across coil
- Voltage proportional to moment on specimen

Measuring magnetic moment of specimen



- Use large coil to apply magnetic field to specimen
- Use a cryostat or furnace to vary temperature of specimen

Response of material to applied magnetic field strength ***H***



- Generally, *M* changes in magnitude as *H* is varied.
- Magnitude of response is called the “magnetic susceptibility” of the material

Response of material to applied magnetic field strength ***H***

- Diamagnetic materials have a very weak negative response

- i.e. they have a small negative magnetic susceptibility

Magnetic susceptibility, χ

- Magnetic susceptibility is sometimes written as

$$\chi = M/H$$

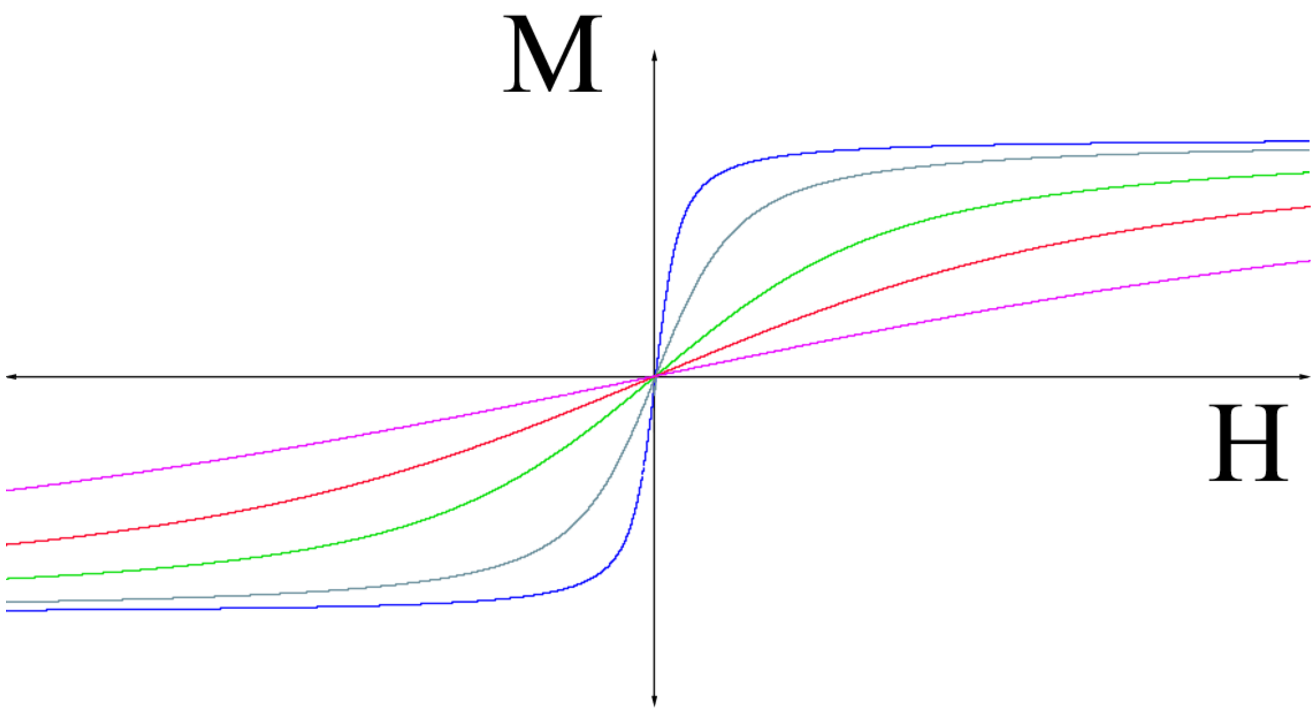
- And sometimes as the slope of *M* vs *H*

$$\chi = dM/dH$$

How does ***M*** respond to ***H***?

- There is a variety of ways that *M* responds to *H*
- Response depends on type of material
- Response depends on temperature
- Response can sometimes depend on the previous history of magnetic field strengths and directions applied to the material

Non-linear responses



$T_1 < T_2 < T_3 < T_4 < T_5$

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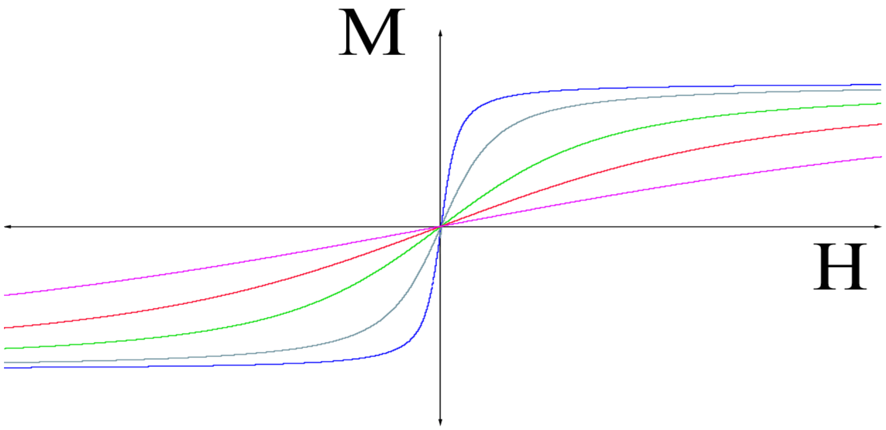
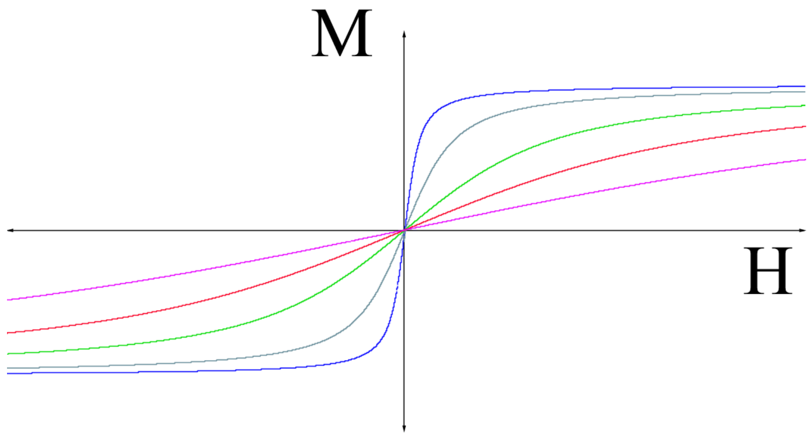
Non-linear responses



$T_1 < T_2 < T_3 < T_4 < T_5$

- Generally, the response of *M* to *H* is non-linear
- Only at small values of *H* or high temperatures is response sometimes linear

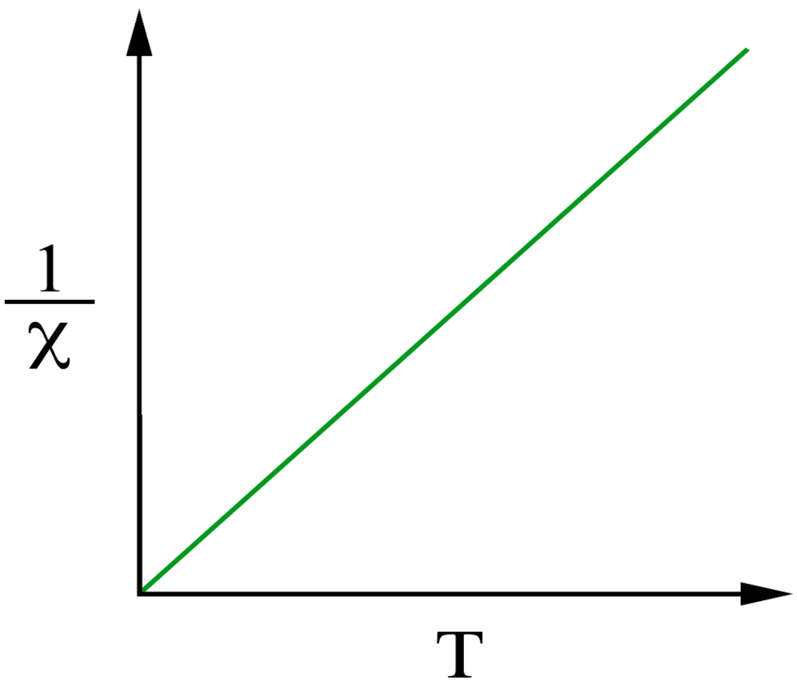
Non-linear responses



$T_1 < T_2 < T_3 < T_4 < T_5$

- ***M*** **tends to saturate at high fields and low temperatures**

Low field magnetic susceptibility



- For some materials, low field magnetic susceptibility is inversely proportional to temperature
- Curie's Law

Materials react to external magnetic fields in three different ways

1) Paramagnetic materials are very weakly attracted by external magnetic fields.

Most materials are paramagnetic.

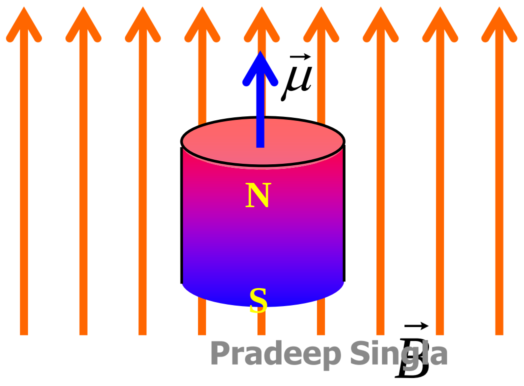
2) Diamagnetic materials are very weakly repelled by external magnetic fields.

3) Ferromagnetic materials are strongly attracted or repelled by external magnetic fields.

How do we understand paramagnetism?

Paramagnetic atoms are like little magnetic dipoles. They experience a torque which aligns them with the external field, then they feel a net force that pulls them into the field.

The magnetic dipole moment results primarily from electron spin and angular momentum.



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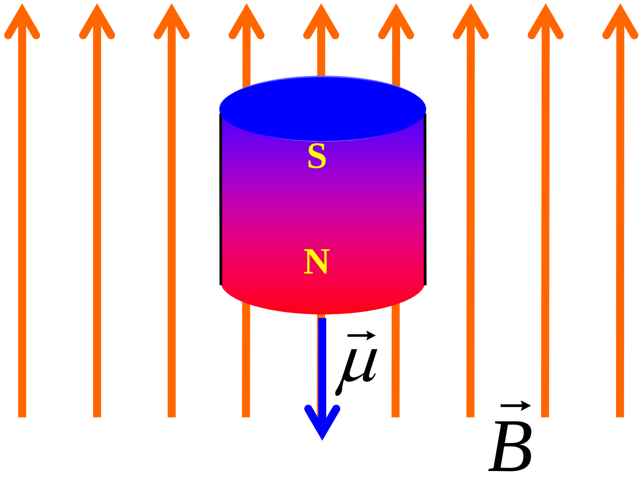
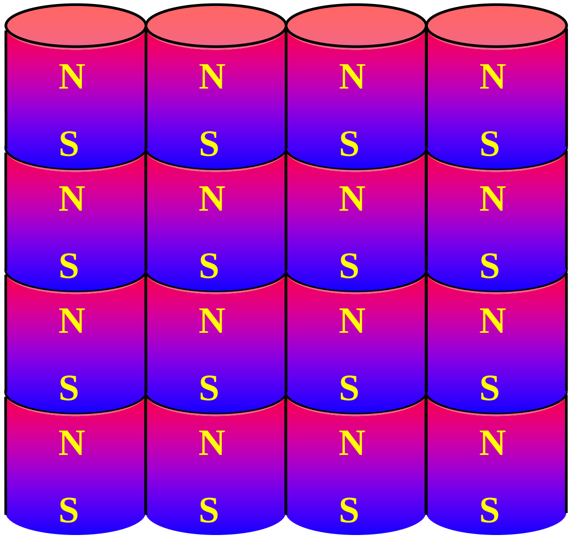
How do we understand diamagnetism?

Diamagnetism is something that is not adequately explained without resorting to quantum mechanics.



How do we understand ferromagnetism?

Domain alignment: If atoms have large magnetic dipole moments, they tend to align with each other much as a collection of magnets tends to align.



How do we understand ferromagnetism?

Thermal disalignment: Heat causes atoms to vibrate, knocking them around and disaligning the dipoles.

The Curie Point

Curie Temperature: When a ferromagnetic material gets hot enough, the domains break down and the material becomes paramagnetic.

Getting Quantitative

We define magnetization as the total magnetic dipole moment per unit volume.

$$\vec{M} = \frac{\sum_{i=1}^{N} \vec{\mu}_i}{Volume}$$

A magnetized object has an internal magnetic field given by the relation:

$$\vec{B}_{\text{int}} = \mu_0 \vec{M}$$

Getting Quantitative

The internal magnetic field can also be expressed in terms of the external magnetic field:

$$\vec{B}_{\text{int}} = \chi \vec{B}_{ext}$$

where χ is called the magnetic susceptibility.

Susceptibilites

paramagnetic	$\chi = +10^{-5}\, to +10^{-3}$
diamagnetic	$\chi \approx -10^{-6}\, to -10^{-4}$
ferromagnetic	$\chi = +10^{+3}\, to +10^{+5}$

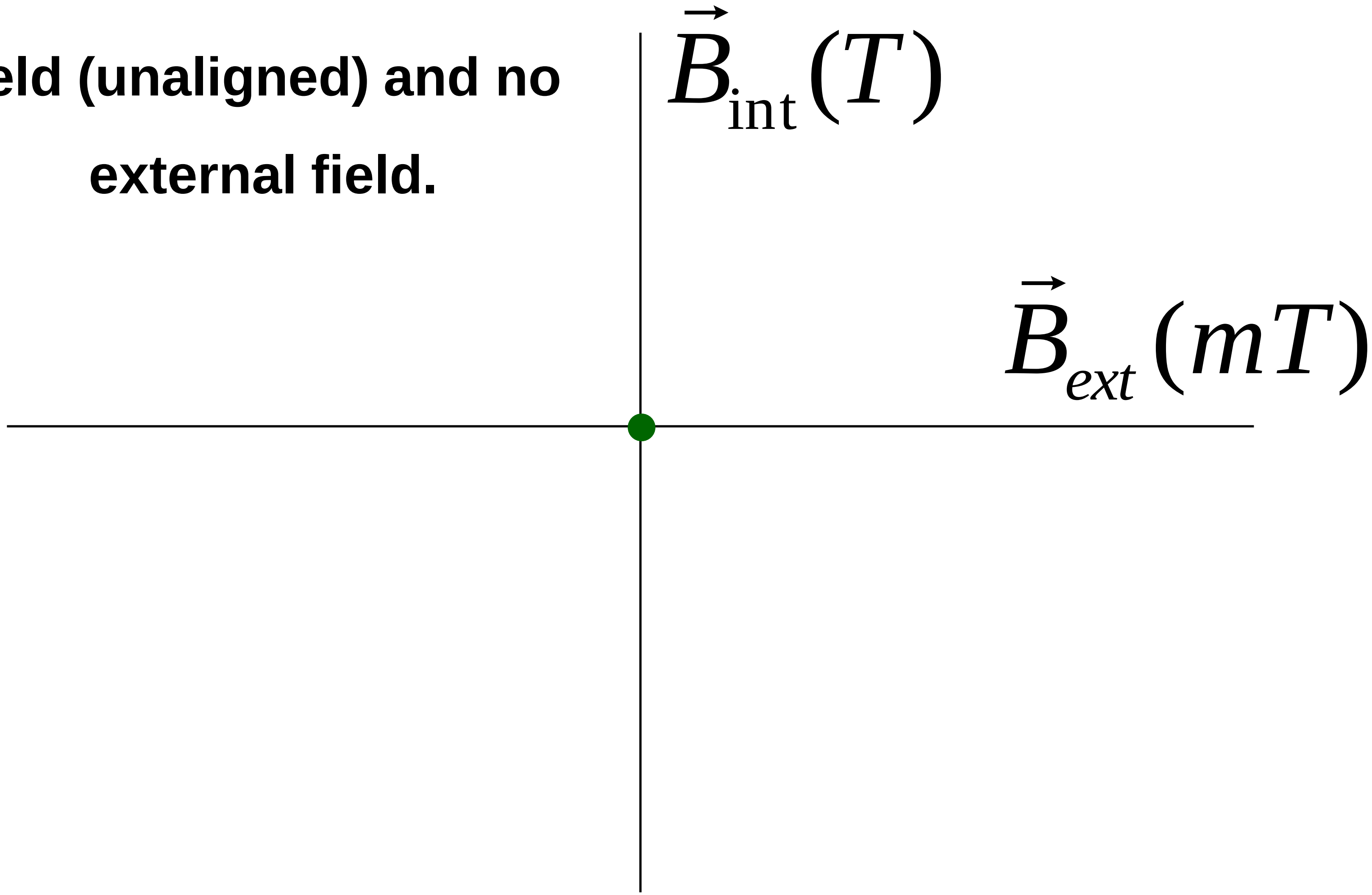
Susceptibilities for Ferromagnetic Materials

Ferromagnetic materials have a “memory.” If we know the external field, we can’t predict the internal field, unless we know the previous history of the sample.

We describe the relationship between internal and external fields by means of a “hysteresis curve.”

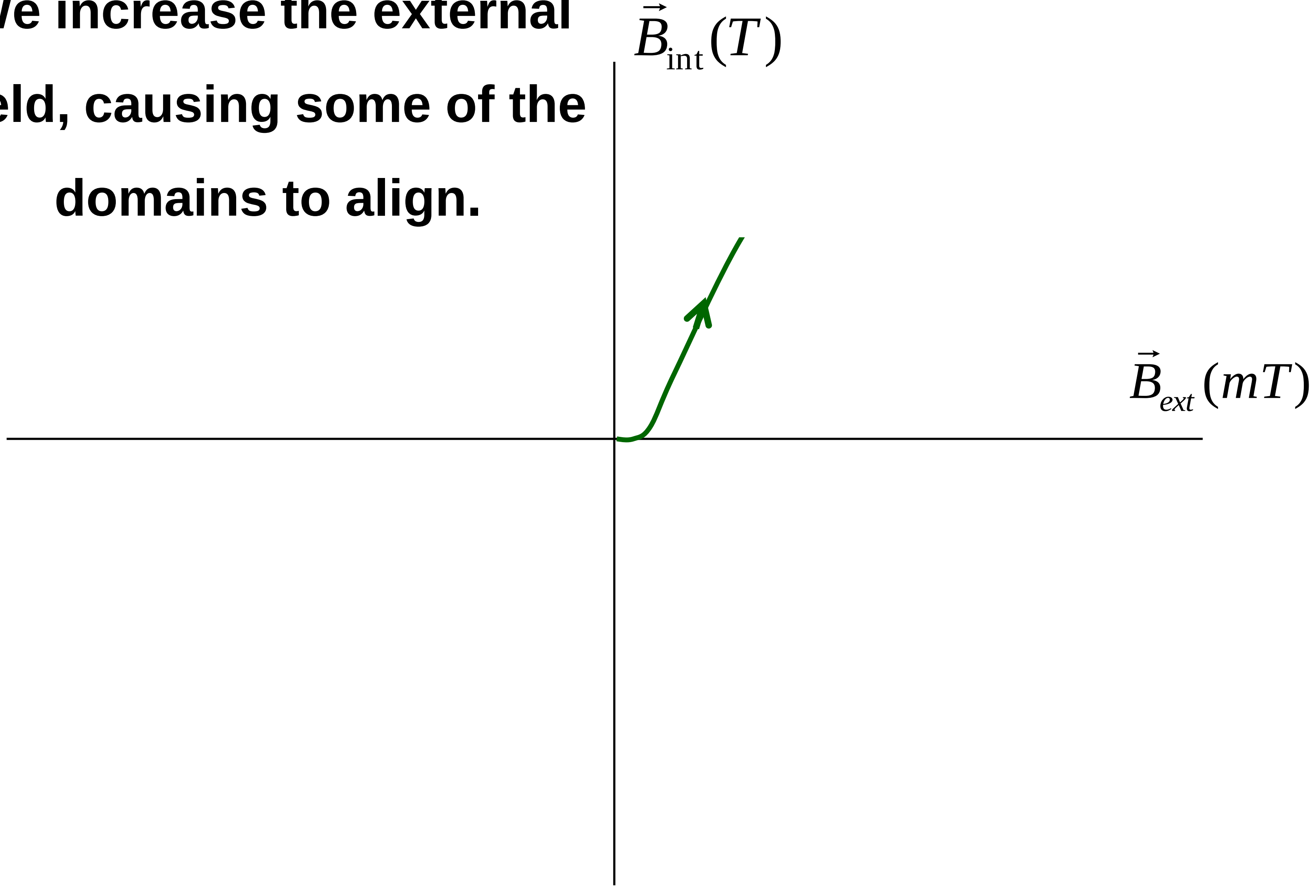
Hysteresis Curve

We start with no internal field (unaligned) and no external field.



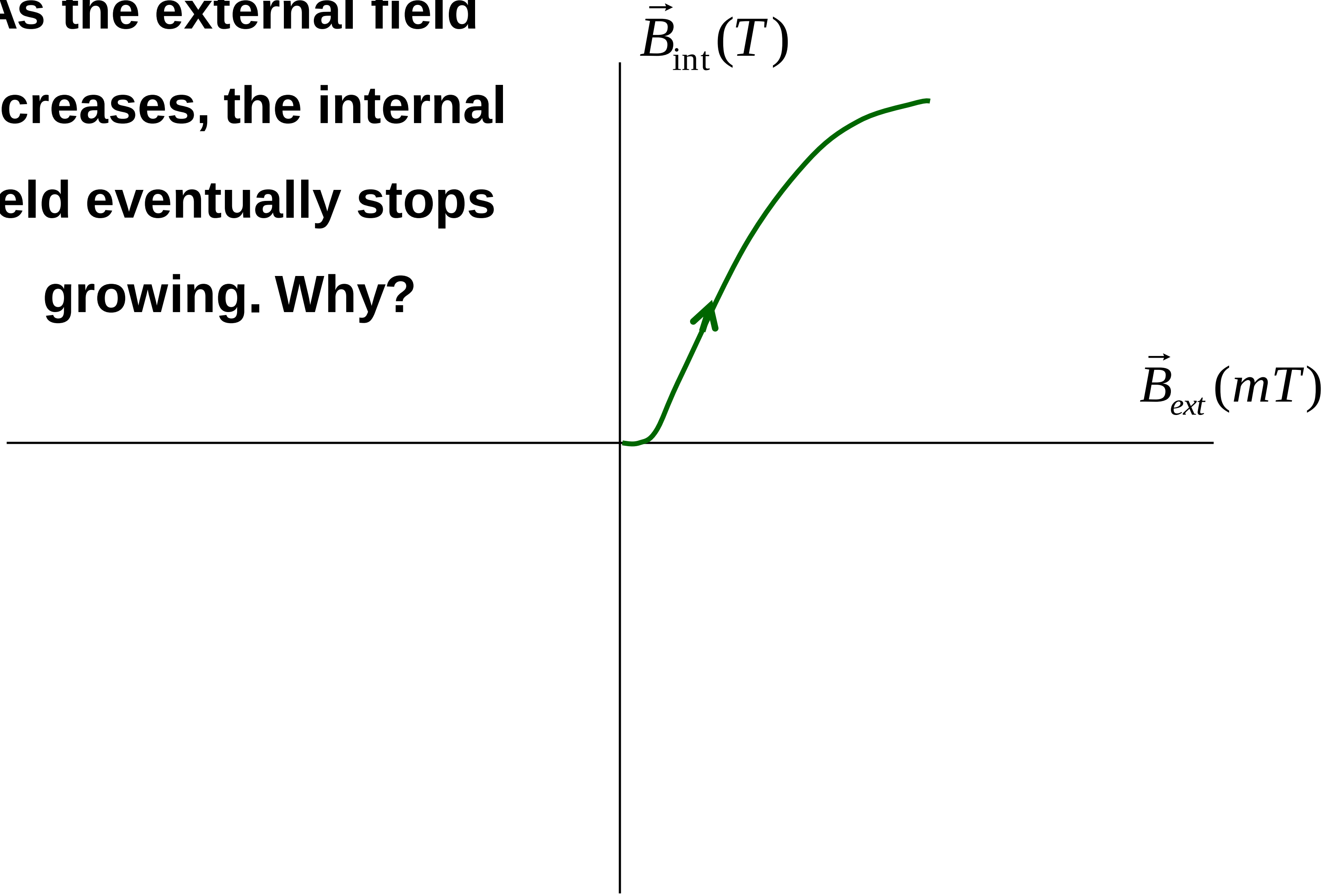
Hysteresis Curve

We increase the external field, causing some of the domains to align.



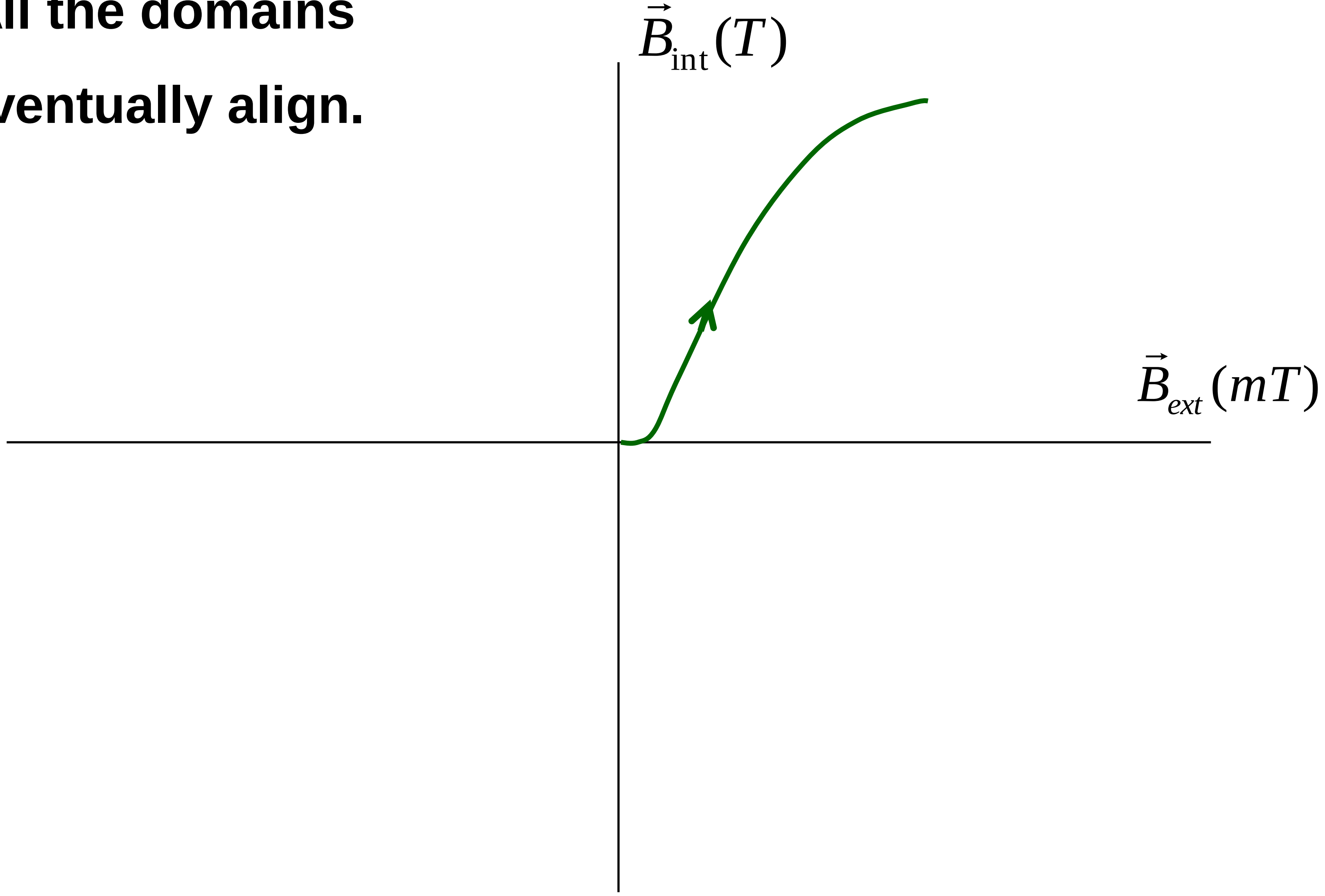
Hysteresis Curve

As the external field increases, the internal field eventually stops growing. Why?



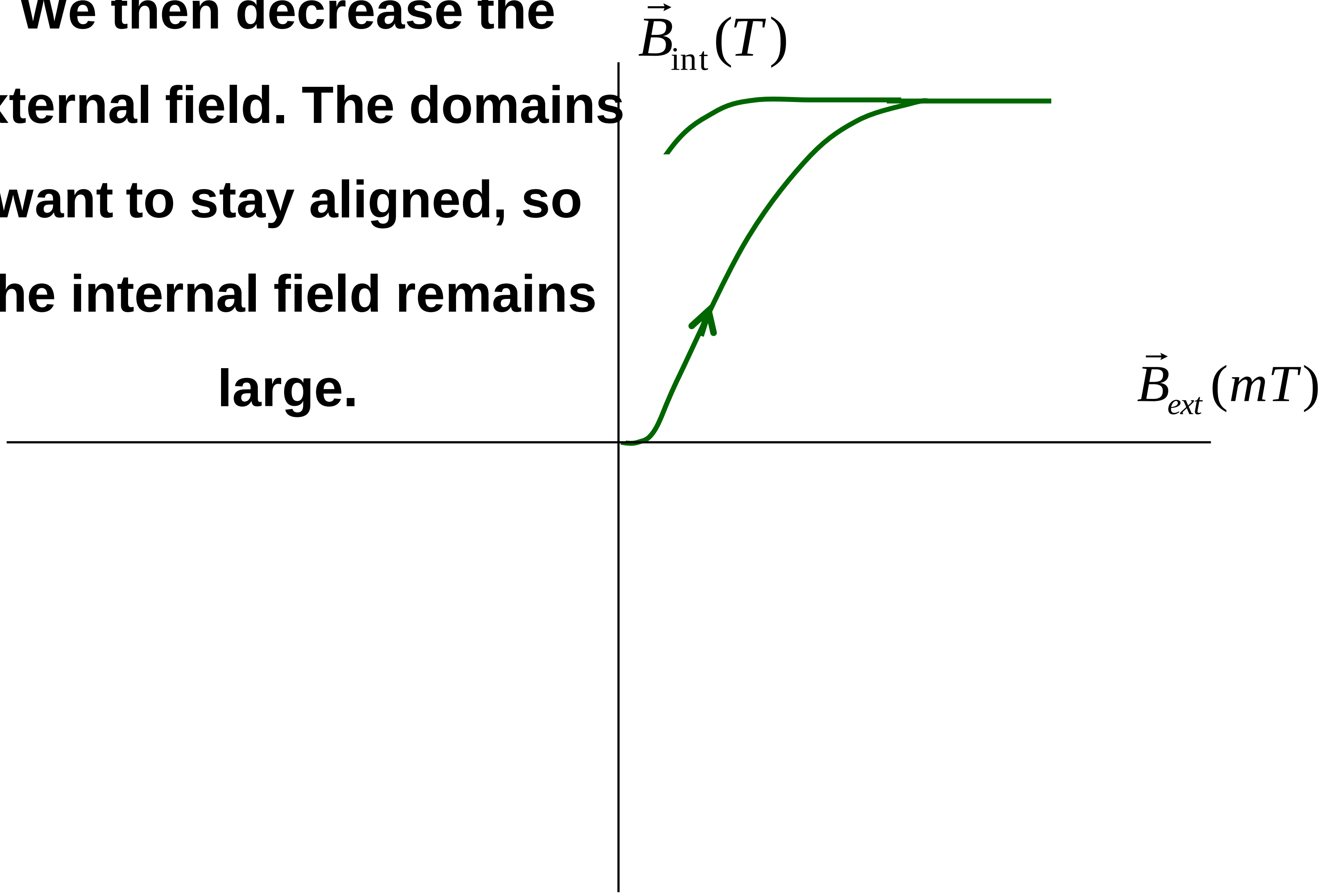
Hysteresis Curve

All the domains eventually align.



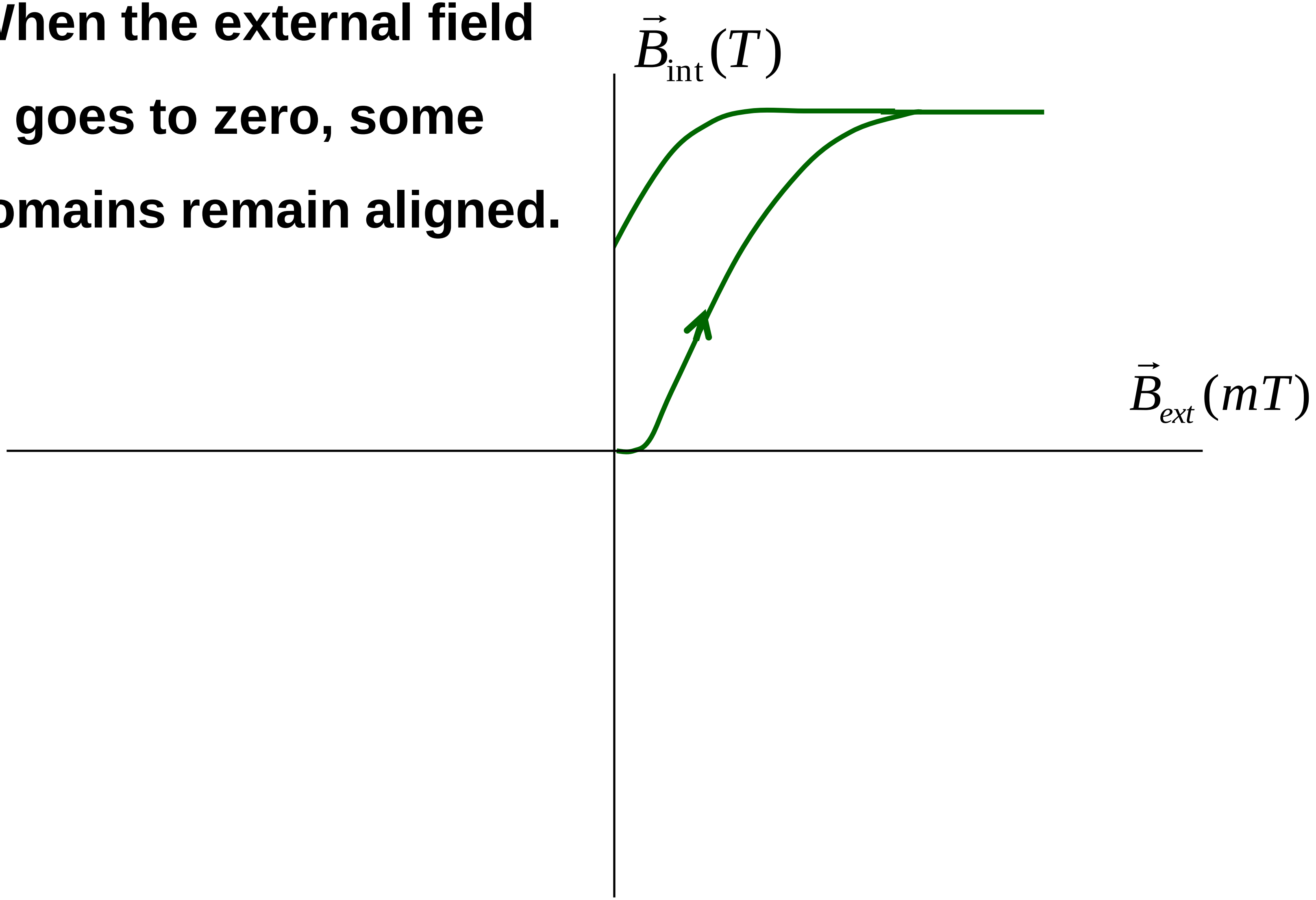
Hysteresis Curve

We then decrease the external field. The domains want to stay aligned, so the internal field remains large.



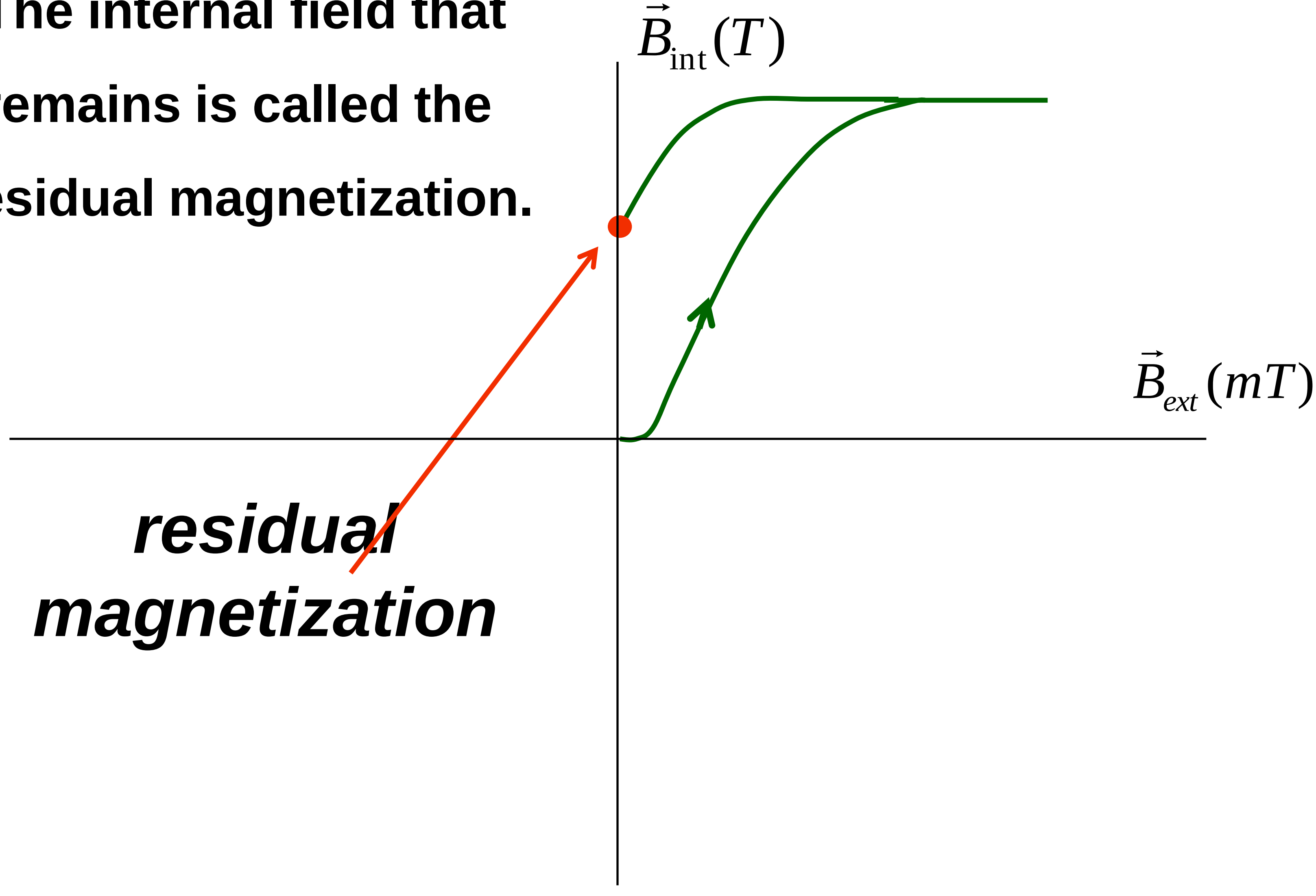
Hysteresis Curve

When the external field goes to zero, some domains remain aligned.



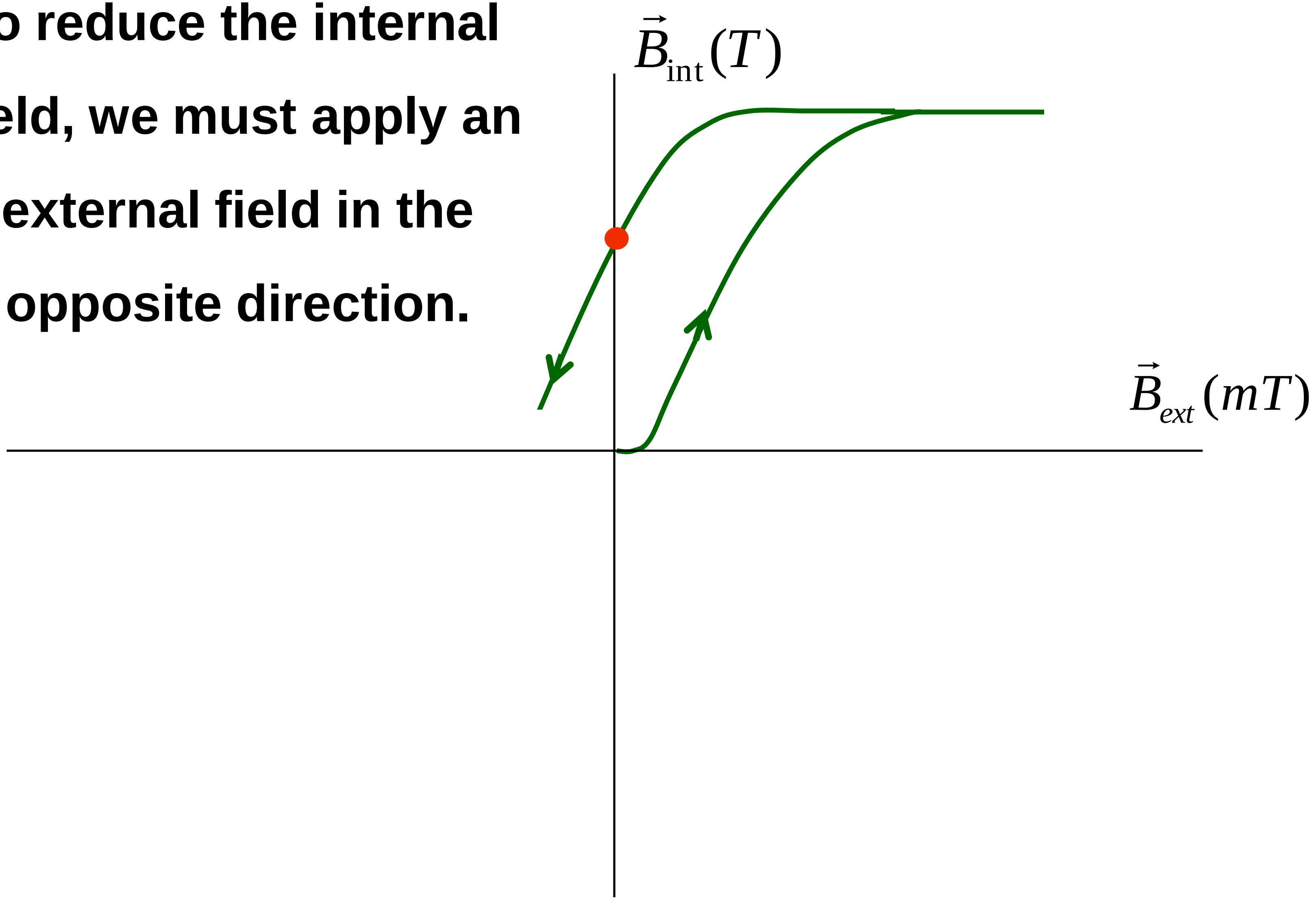
Hysteresis Curve

The internal field that remains is called the residual magnetization.



Hysteresis Curve

To reduce the internal field, we must apply an external field in the opposite direction.



Hysteresis Curve

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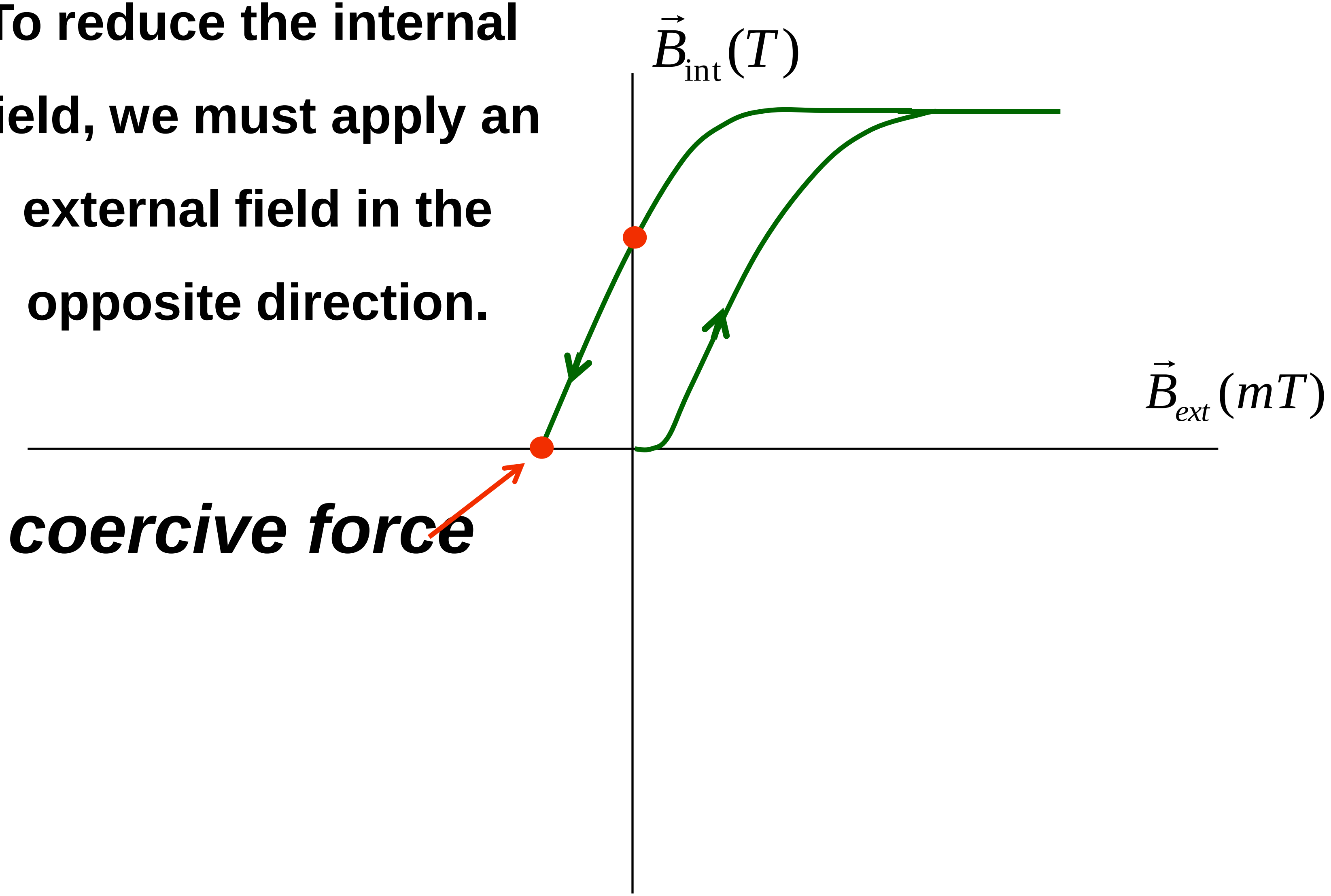
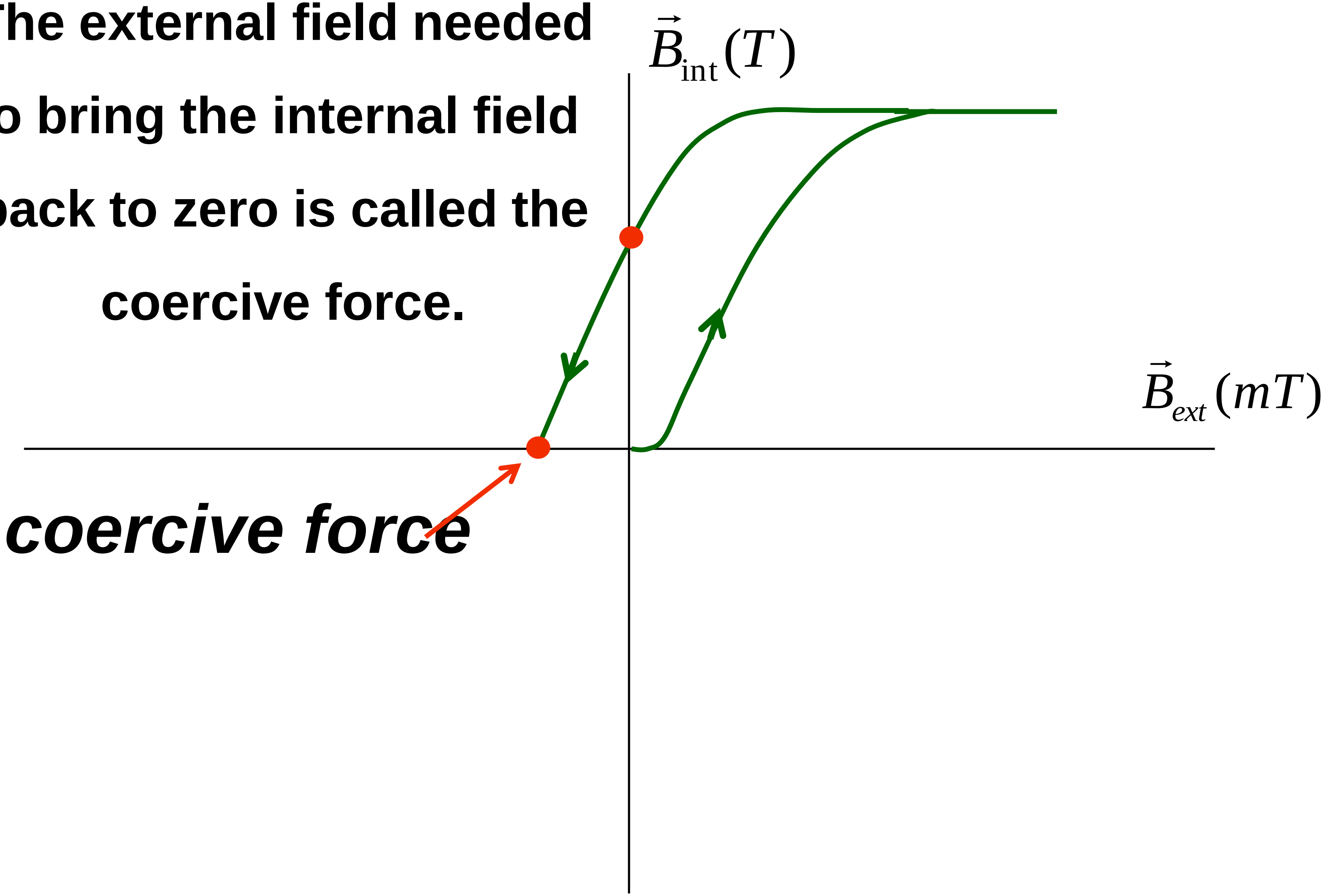


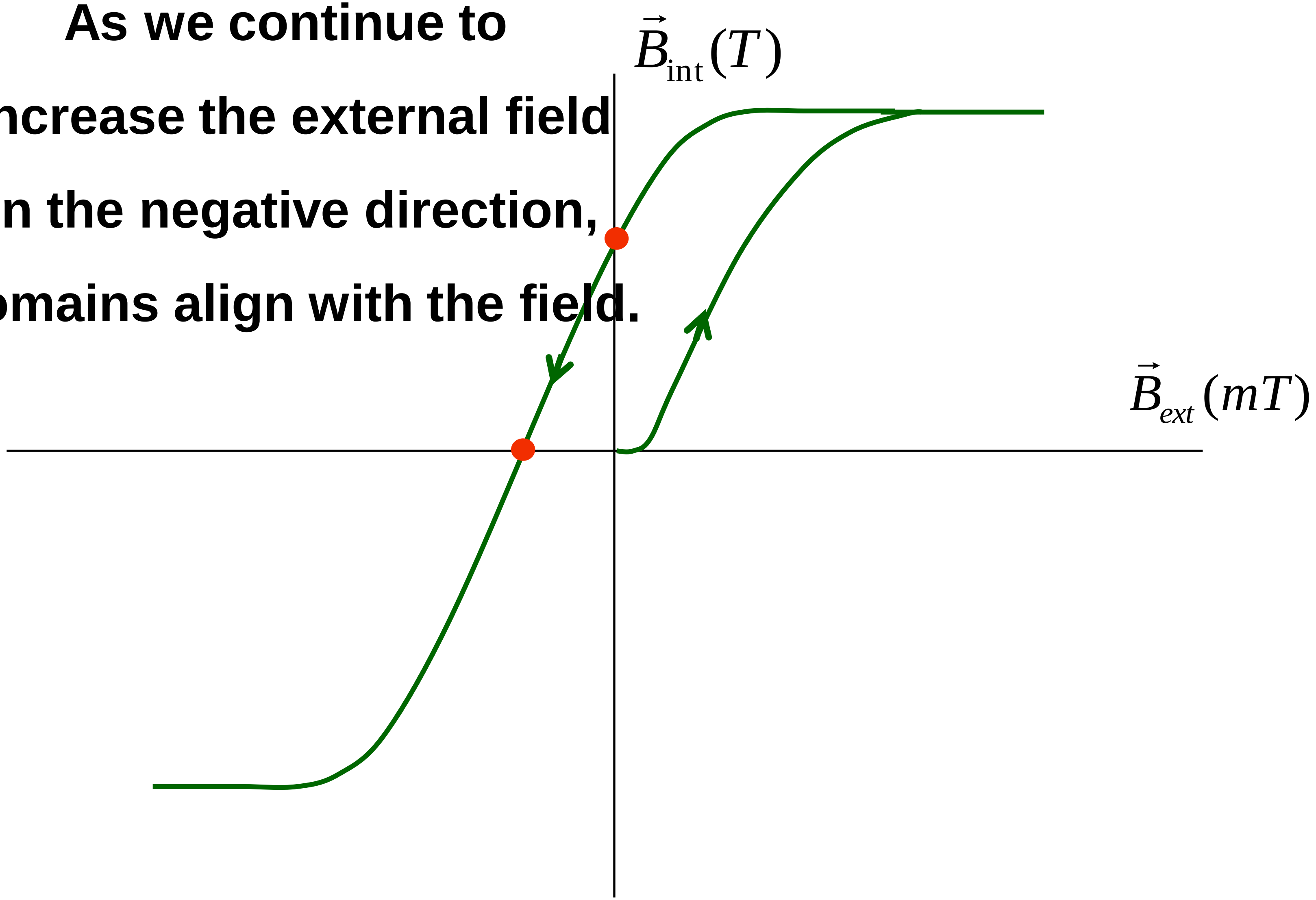
Hysteresis Curve

The external field needed to bring the internal field back to zero is called the coercive force.



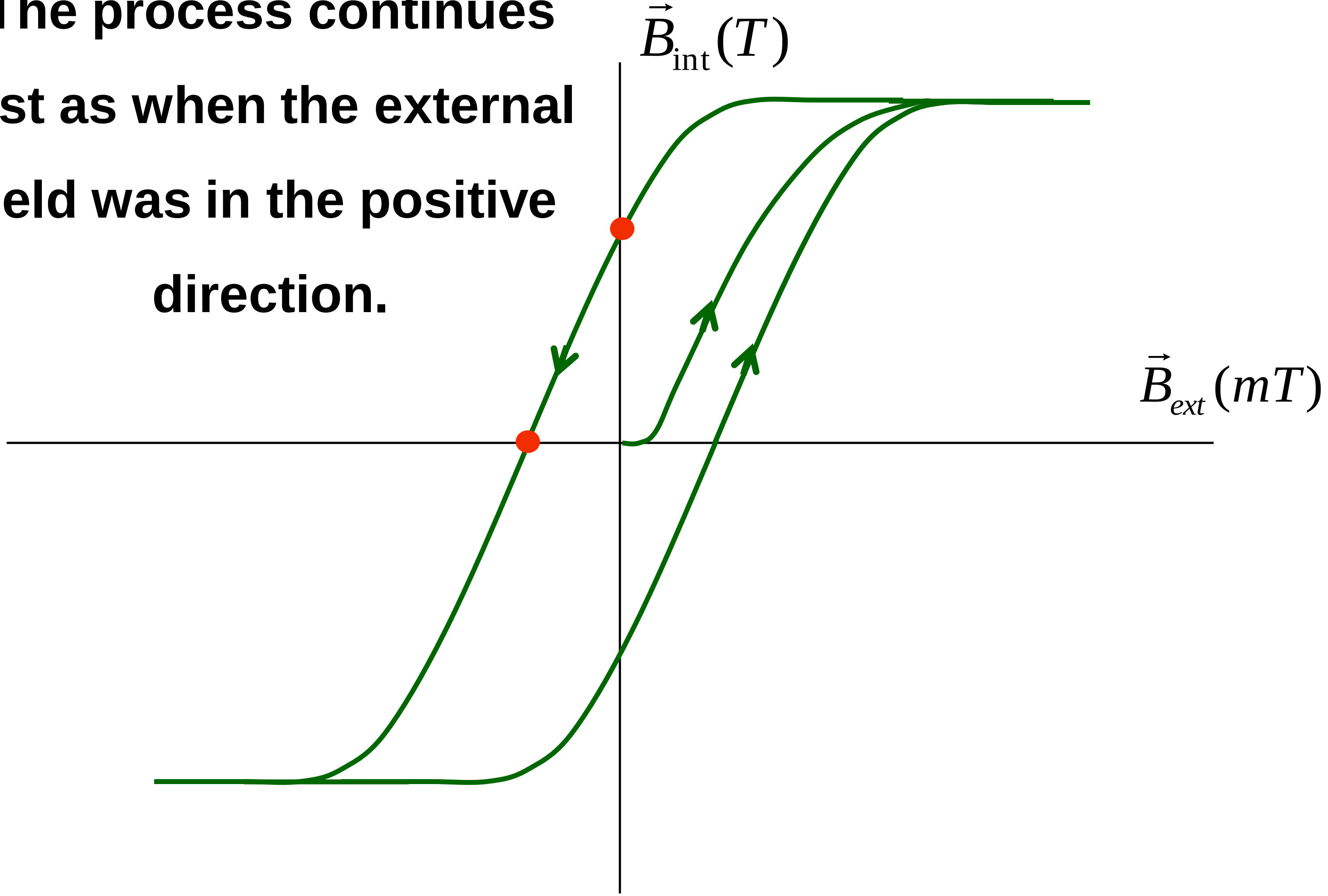
Hysteresis Curve

As we continue to increase the external field in the negative direction, domains align with the field.



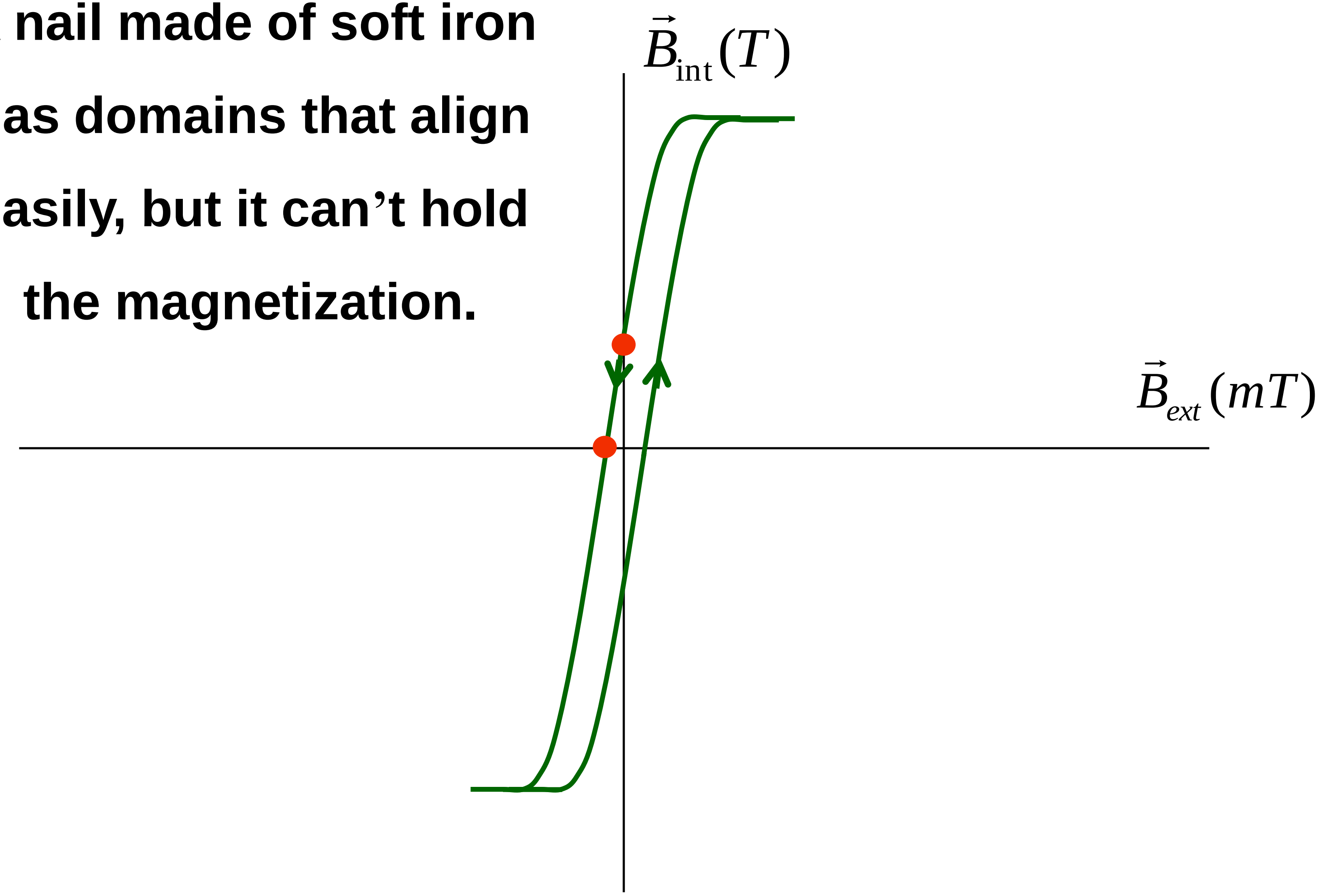
Hysteresis Curve

The process continues just as when the external field was in the positive direction.



Soft Iron

A nail made of soft iron has domains that align easily, but it can't hold the magnetization.



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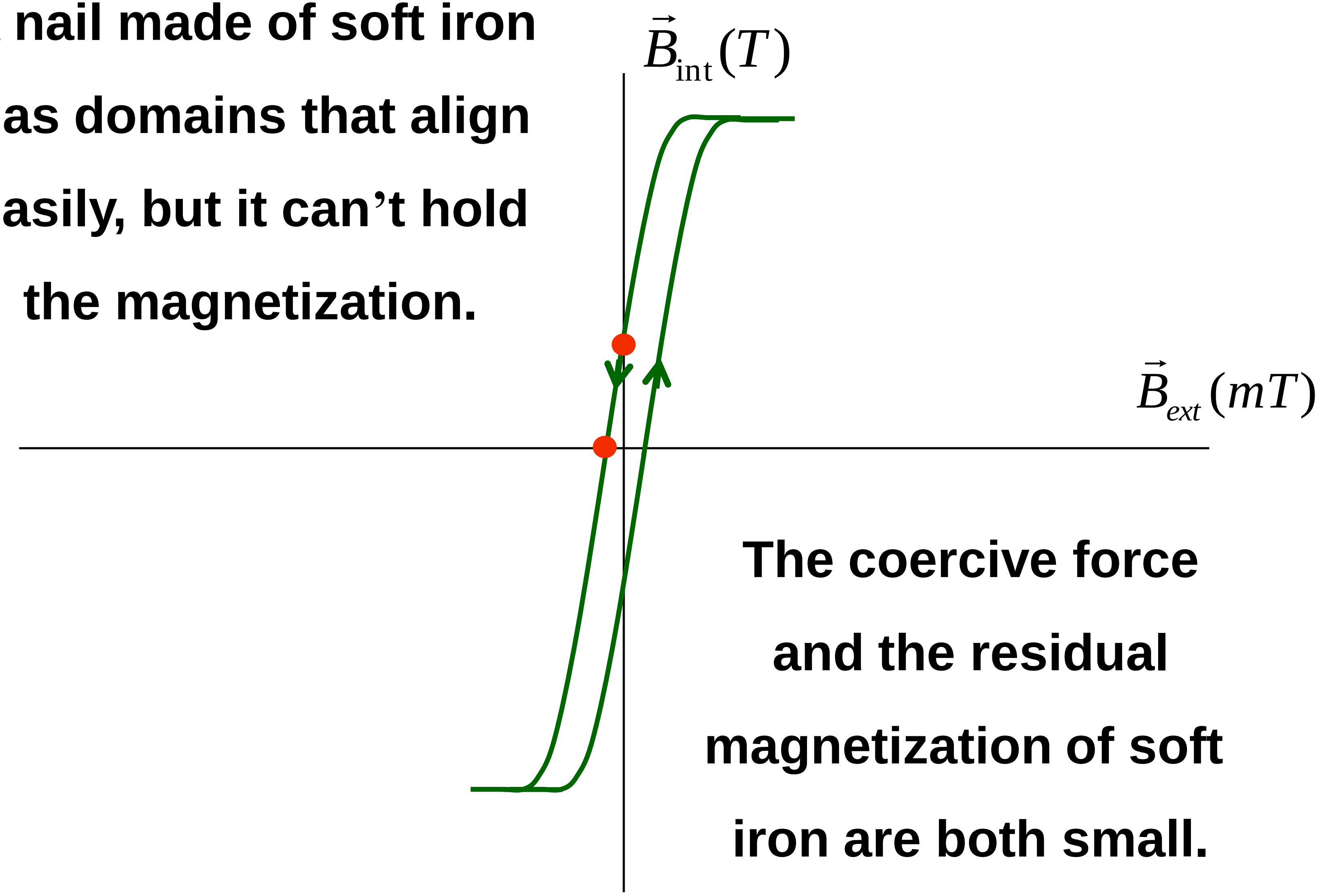
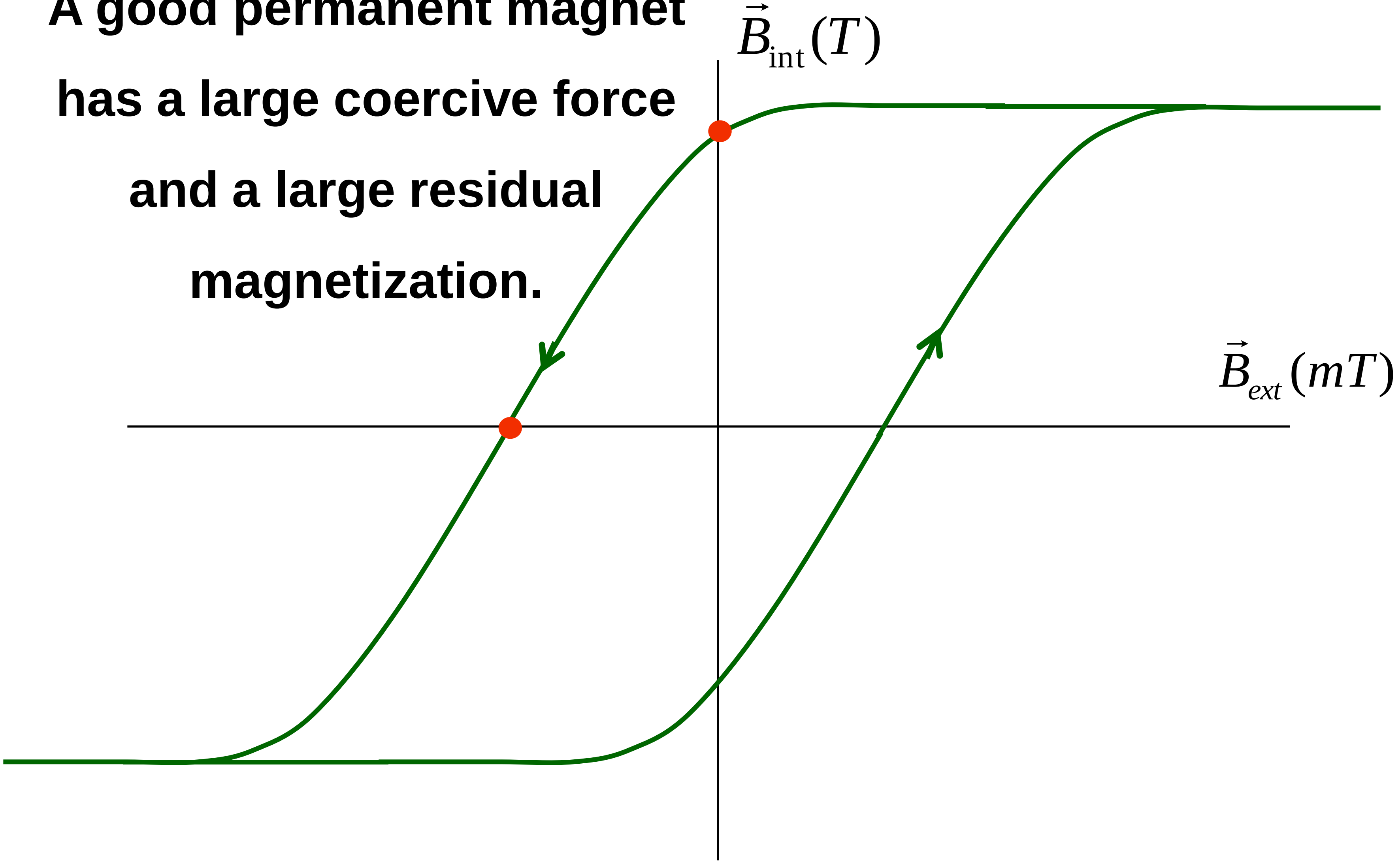
A nail made of soft iron has domains that align easily, but it can’t hold the magnetization.



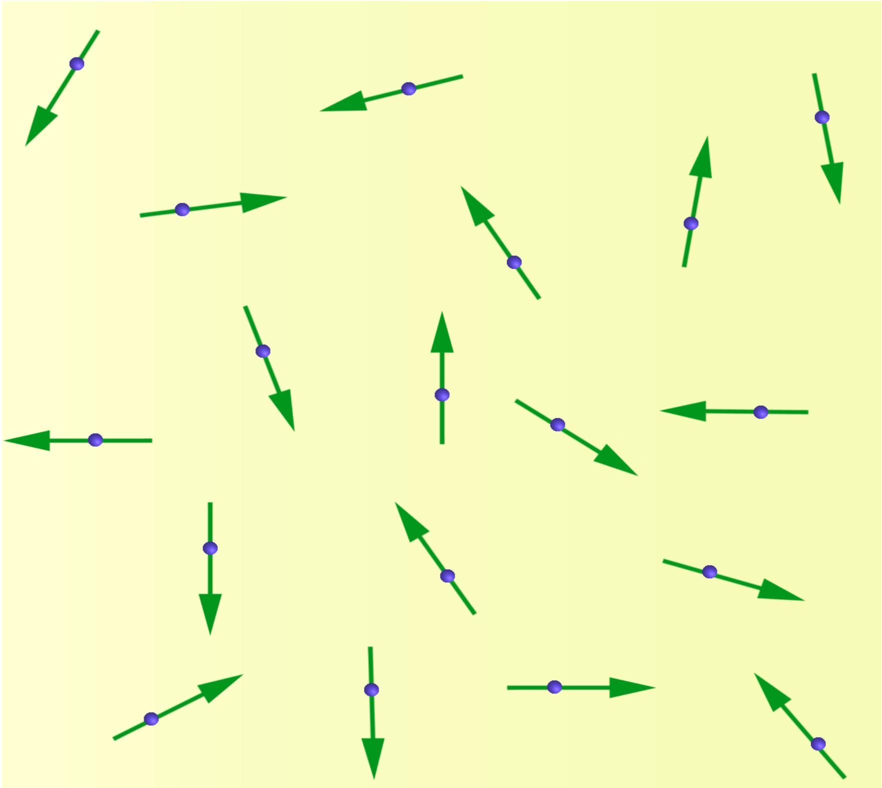
The coercive force and the residual magnetization of soft iron are both small.

Good Permanent Magnet

A good permanent magnet has a large coercive force and a large residual magnetization.

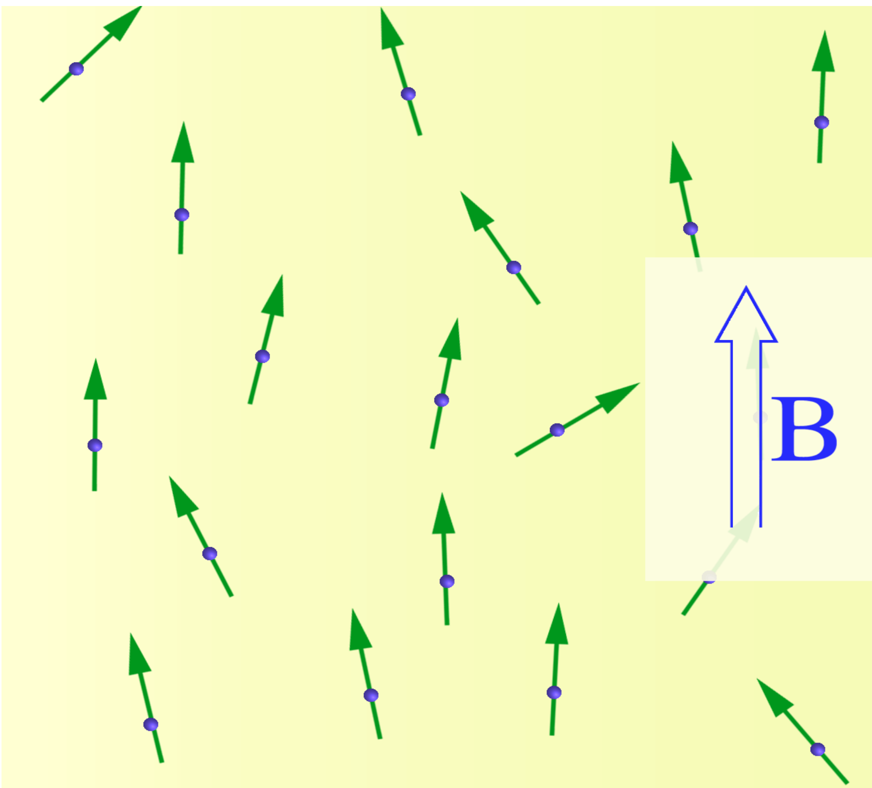


Paramagnetic gas



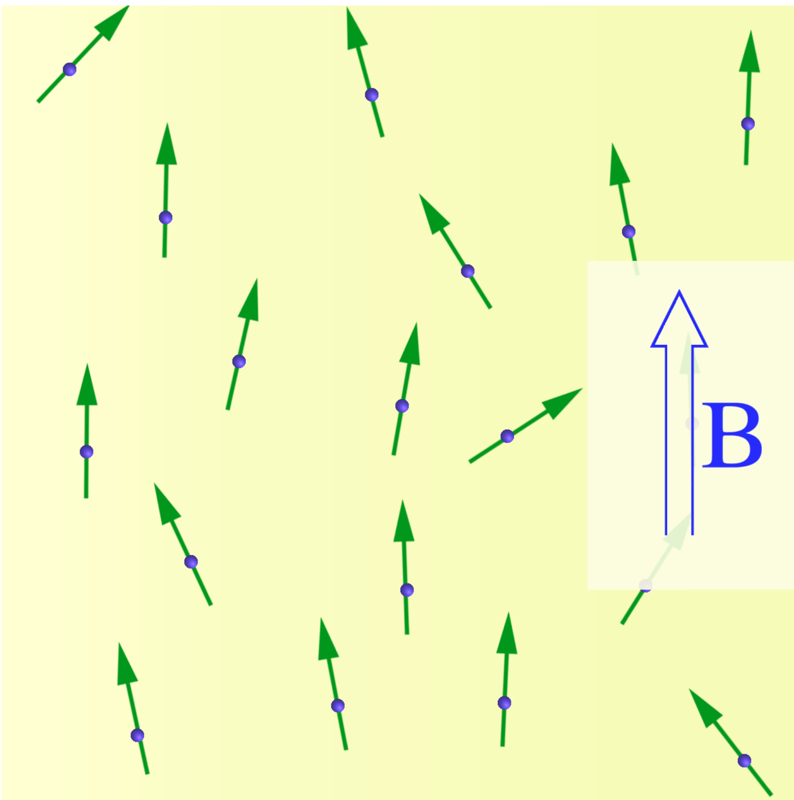
- Imagine a classical gas of molecules each with a magnetic dipole moment
- In zero field the gas would have zero magnetization

Paramagnetic gas



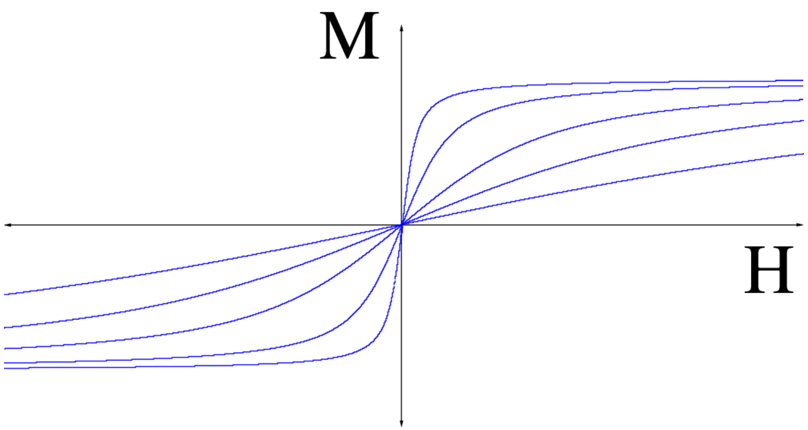
- Applying a magnetic field would tend to orient the dipole moments
- Gas attains a magnetization

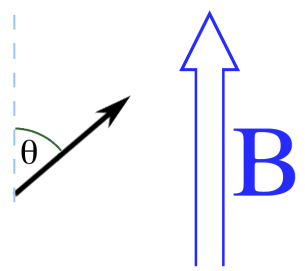
Paramagnetic gas



- Very high fields would saturate magnetization
- Heating the gas would tend to disorder the moments and hence decrease magnetization

Paramagnetic gas

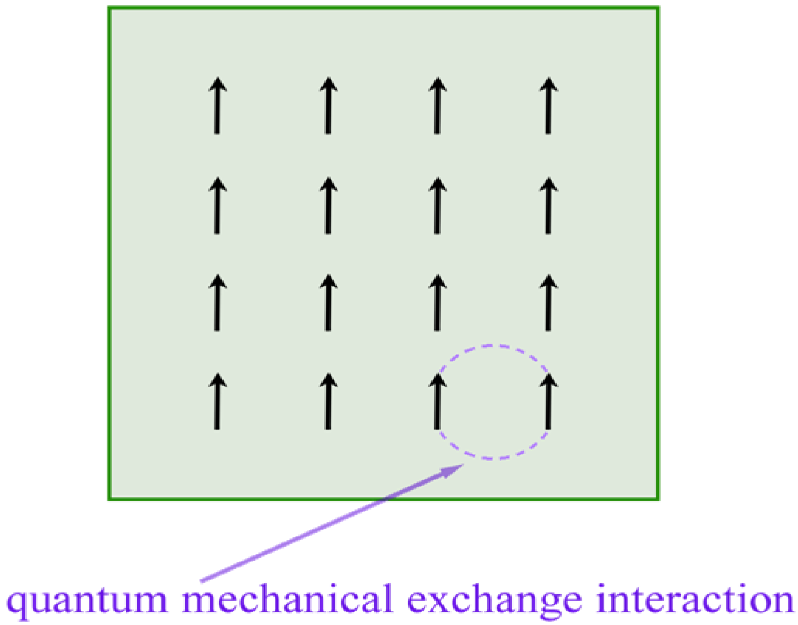




$E = -m\, B \cos[\theta]$

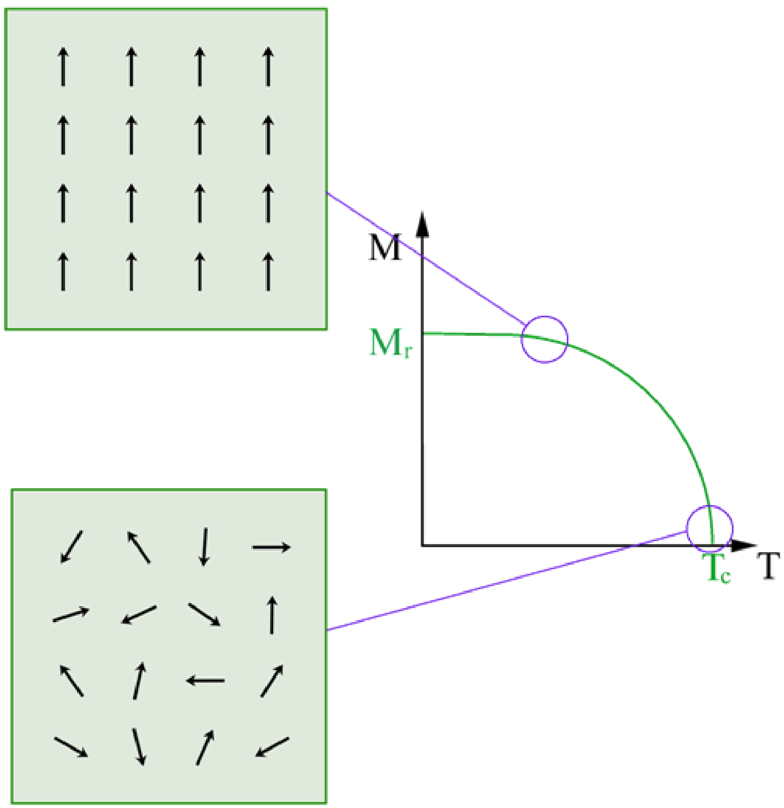
- Theoretical model
- Non-interacting moments
- Boltzmann statistics
- Dipole interaction with *B*
- Yields good model for many materials
- Examples: ferrous sulfate crystals, ionic solutions of magnetic atoms

Ferromagnetism



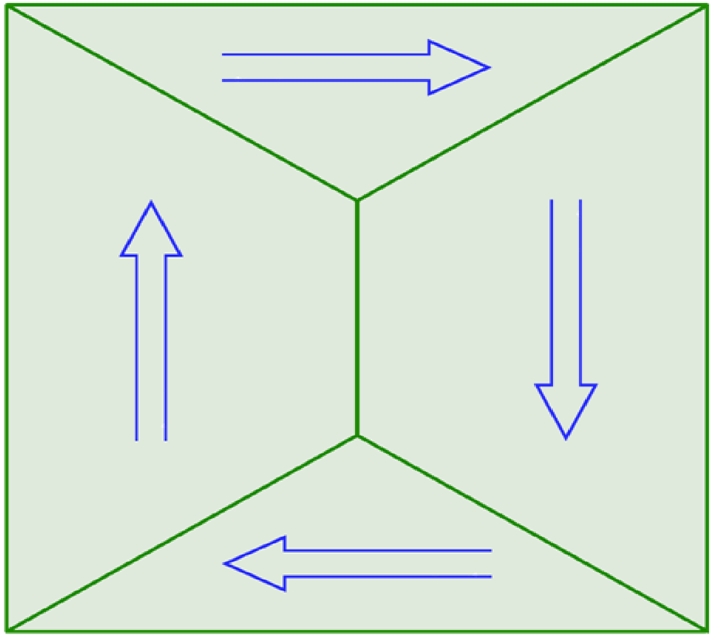
- Materials that retain a magnetization in zero field
- Quantum mechanical exchange interactions favour parallel alignment of moments
- Examples: iron, cobalt

Ferromagnetism



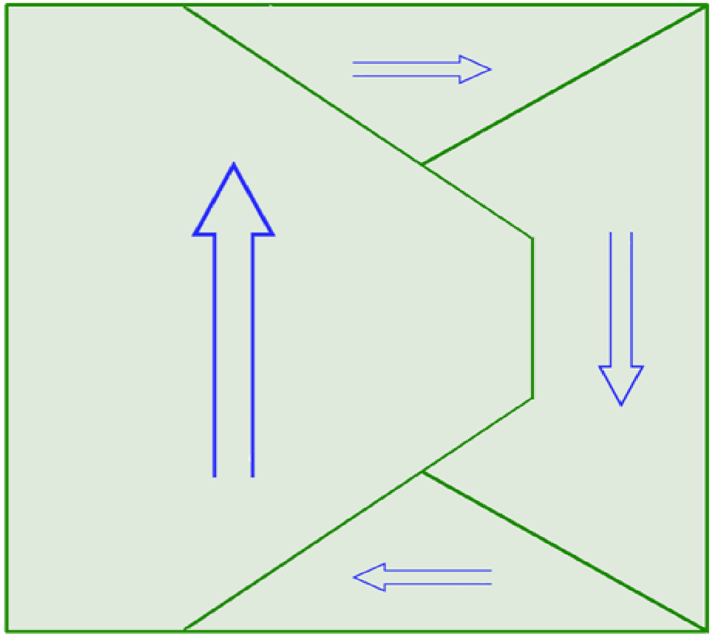
- Thermal energy can be used to overcome exchange interactions
- Curie temp is a measure of exchange interaction strength
- Note: exchange interactions much stronger than dipole-dipole interactions

Magnetic domains



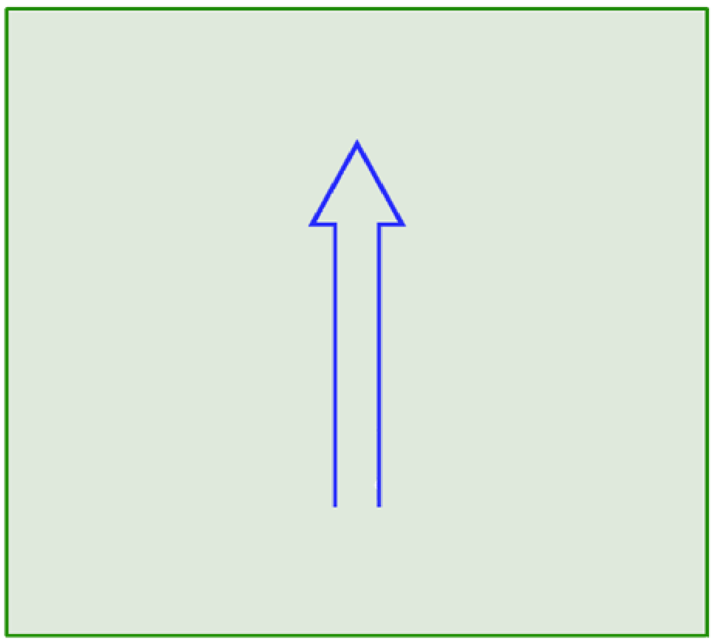
- Ferromagnetic materials tend to form magnetic domains
- Each domain is magnetized in a different direction
- Domain structure minimizes energy due to stray fields

Magnetic domains



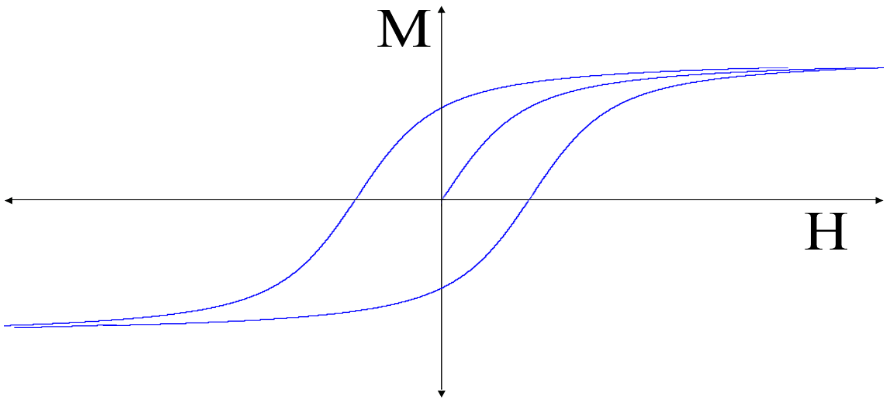
- Applying a field changes domain structure
- Domains with magnetization in direction of field grow
- Other domains shrink

Magnetic domains



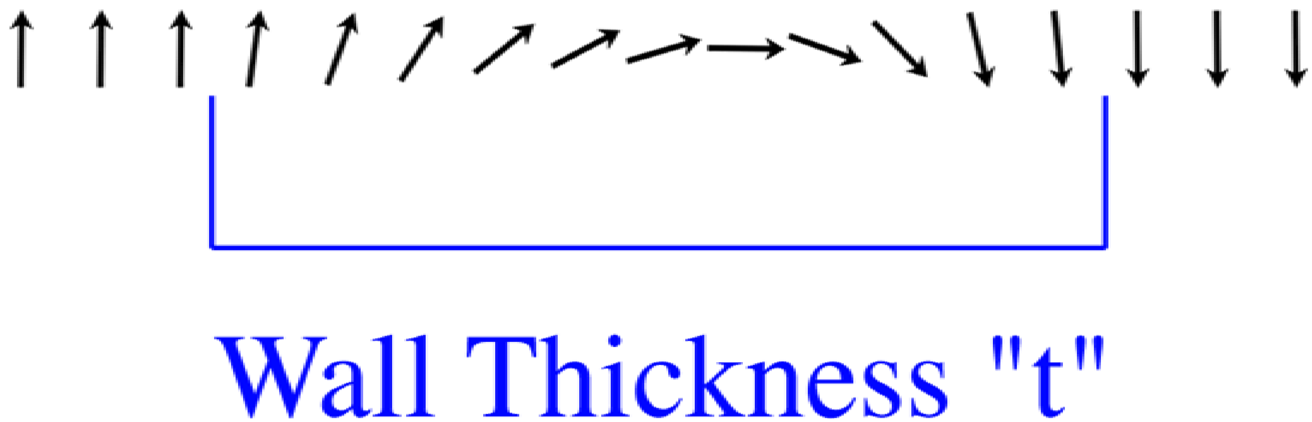
- Applying very strong fields can saturate magnetization by creating single domain

Magnetic domains



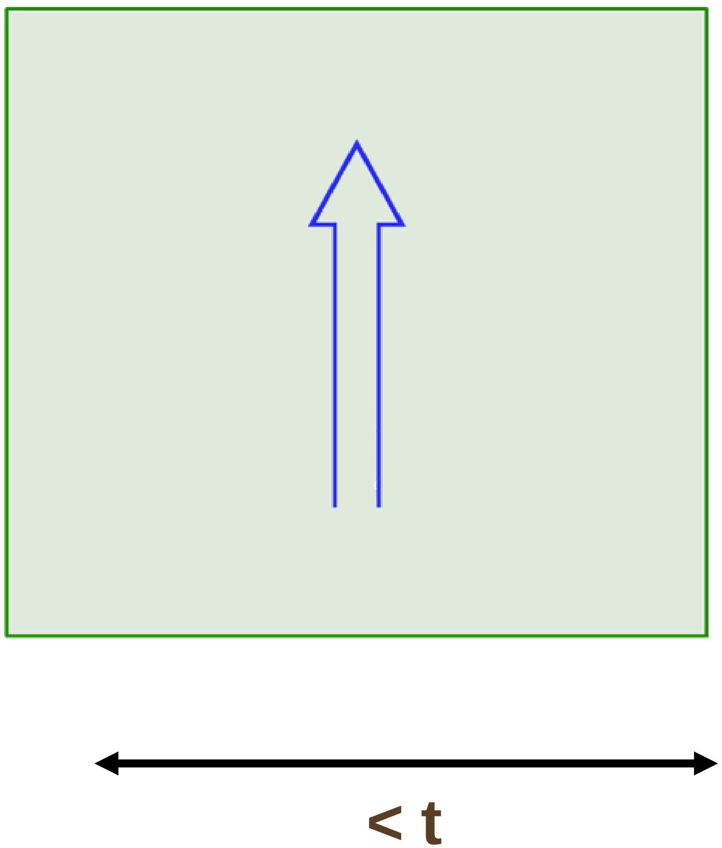
- Removing the field does not necessarily return domain structure to original state
- Hence results in magnetic hysteresis

Magnetic domain walls



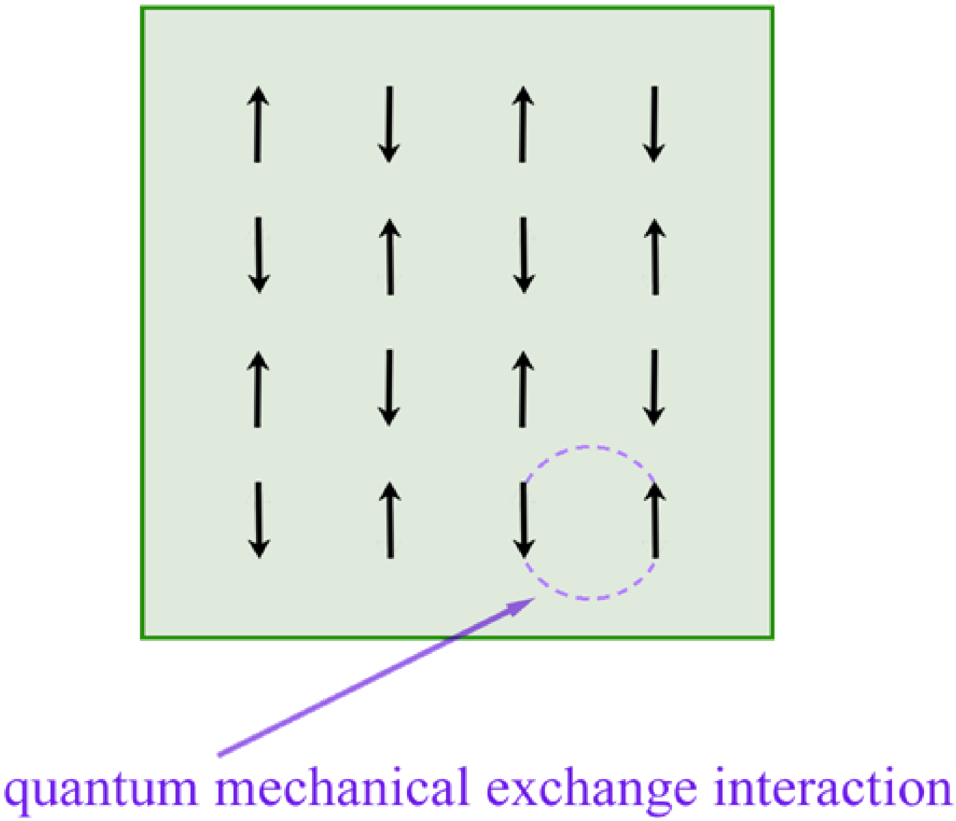
Wall thickness, t, is typically about 100 nm

Single domain particles



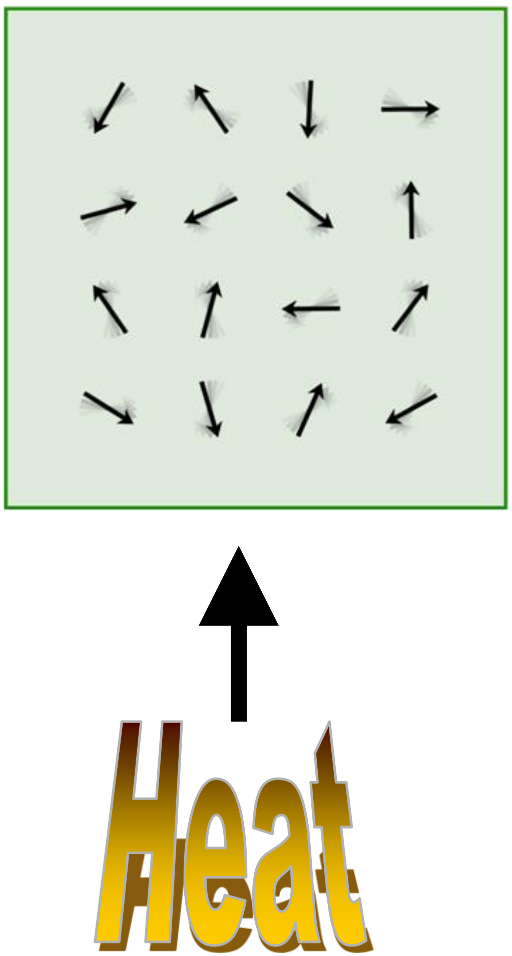
- Particles smaller than "t" have no domains

Antiferromagnetism



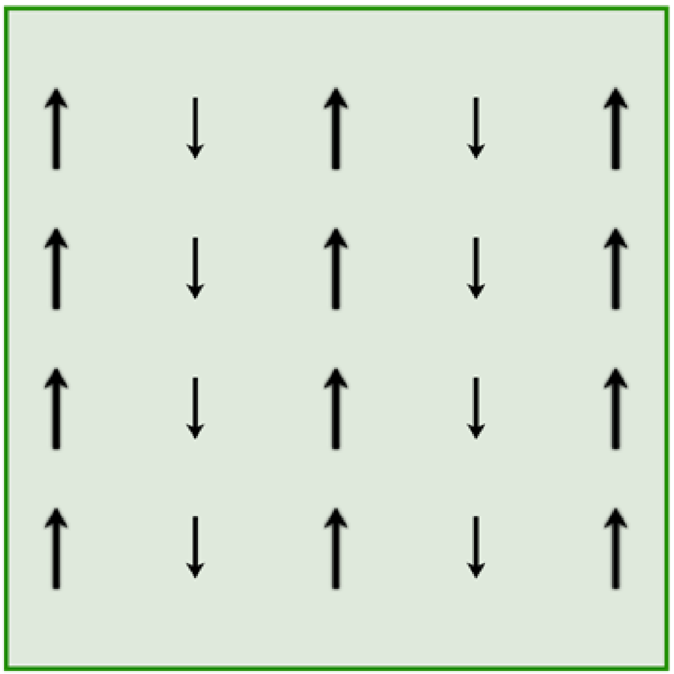
- In some materials, exchange interactions favour antiparallel alignment of atomic magnetic moments
- Materials are magnetically ordered but have zero remnant magnetization and very low χ
- Many metal oxides are antiferromagnetic

Antiferromagnetism



- Thermal energy can be used to overcome exchange interactions
- Magnetic order is broken down at the Néel temperature (c.f. Curie temp)

Ferrimagnetism



- Antiferromagnetic exchange interactions
- Different sized moments on each sublattice
- Results in net magnetization
- Example: magnetite, maghemite

Magnetization Quantified

- Approach: define Amperian (bound) current
  1) Associated with bound charges of electrons in atoms locked into lattice structure of a material
  2) Magnetic dipole moment of each individual charge

$$\vec{m}_i = I_b d\vec{S}\ [\mathrm{A \cdot m^2}]$$

  3) For n dipoles per unit volume

$$\vec{m}_{total} = \sum_{i=1}^{n\Delta V} \vec{m}_i\ [\mathrm{A \cdot m^2}]$$

  4) Magnetization: magnetic dipole moment per unit vol.

$$\vec{M} = \lim_{\Delta V \to 0} \left(\frac{1}{\Delta V} \vec{m}_{total}\right)\ [\mathrm{A/m}]$$

Pradeep Singla

Bound versus Free Current Forms of A.C.L.

- Mathematematic Expressions:

Bound versus Free

$$I_b = \oint \vec{M} \cdot d\vec{L} \text{ [A]}$$

where $I_b \equiv$ bound current

$$I = \oint \vec{H} \cdot d\vec{L} \text{ [A]}$$

where $I \equiv$ free current

- Note:

I_b depends on the number and alignment of the miniature magnetic dipoles along the closed path

Bound & Free Currents Combined

- Total Current

$$I_T = I_b + I = \oint (\vec{B}/\mu_0) \cdot d\vec{L} \quad \text{(A.C.L.)}$$

(total)=(bound)+(free) (since $\vec{H}=\vec{B}/\mu_0$ in free space)

- Free Current

$$I = I_T - I_b = \oint (\vec{B}/\mu_0) \cdot d\vec{L} - \oint \vec{M} \cdot d\vec{L}$$

$$\therefore I = \oint (\vec{B}/\mu_0 - \vec{M}) \cdot d\vec{L} \Rightarrow \vec{H} = \vec{B}/\mu_0 - \vec{M} \quad \text{[A/m]}$$

- General Relation for **B** versus **H**

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad \text{[A/m]}$$

Linear Isotropic Magnetic Media (e.g., paramagnetic, diamagnetic)

- Magnetization $\vec{M} = \chi_m \vec{H}$

where $\chi_m \equiv$ magnetic susceptibility [unitless constant]

- Magnetic Flux Density

$$\begin{aligned} \vec{B} &= \mu_0(\vec{H} + \vec{M}) \\ &= \mu_0(\vec{H} + \chi_m \vec{H}) = \mu_0(1 + \chi_m)\vec{H} \\ &= \mu_r \mu_0 \vec{H} = \mu \vec{H} \end{aligned}$$

where $\mu_r = 1 + \chi_m$

$\equiv$ relative permeability

and $\mu = \mu_r \mu_0 =$ permeability [H/m]

Nonlinear & Anisotropic Materials (e.g., ferromagnetic)

- If magnetization (**M**) responds nonlinearly to an imposed magnetic field (**H**) such as for a ferromagnetic polycrystalline material
  - relation $\vec{B} = \mu_0(\vec{H} + \vec{M})$ still applies, but. . .
  - the parameters χ_m and μ_r will not be constant material properties since **H** vs. **M** is nonlinear
- If linear and homogeneous, but anisotropic as for a ferromagnetic single crystal then

$$\vec{B} = [\mu]\vec{H} \quad \text{where } [\mu] = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yx} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \text{ is a } 3\times 3 \text{ matrix}$$

Alternative Derivation

We now wish to describe magnetization phenomena and their effects on the total magnetic field in materials. Similar to what we did for the polarization vector **P**, we introduce the *magnetic moment per unit volume*, also called magnetization density, or simply magnetization.

$$\mathbf{M} = \frac{\Sigma_i \mathbf{m}_i}{V}$$

MAGNETIZATION

$$[\mathbf{M}] = \frac{A}{m}$$

We can always think of **M** as generated by electric currents inside the material. These will be different from those considered so far, i.e. not limited to the motion of free charges. However, as all physical currents will contribute to the total magnetic field **B**, we can expect to write the total field as generated by the total current density:

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J}_{\mathbf{free}} + \mathbf{J}_{\mathbf{M}})$$

J_M : MAGNETIC CURRENT DENSITY

where $\mathbf{J}_\mathbf{M}$ represents the electric currents which generate **M** in the material, and $\mathbf{J}_{\mathbf{free}}$ represents the electric currents due to the movement of free charges.

An expression for the magnetization current density $\mathbf{J}_M$

We already know that the free current vector field **J** generates a vector potential:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d^3r'$$

We now want to add to this the contribution from the local magnetic dipole moments present in the material considered.

Since the vector potential at point **r** from a magnetic moment **m** is:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\mathbf{r}}{r^3}$$

The magnetization **M** represents the magnetic moment density. Thus, we can write, for the **A** contribution due to magnetization, the following space integral:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\mathbf{M}(\mathbf{r}')\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} d^3r'$$

VECTOR POTENTIAL ASSOCIATED TO THE MAGNETIZATION

Where the volume V includes the material considered.
This can be also written as:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_V \mathbf{M}(\mathbf{r}')\times\nabla'\frac{1}{|\mathbf{r}-\mathbf{r}'|} d^3r'$$

∇' = Gradient w.r.t. the **r'** variables.

Now notice that, in general, we have:

$$\nabla\times(f\,\mathbf{M}) = \nabla f\times\mathbf{M} + f\,\nabla\times\mathbf{M}$$

f, **M** = any scalar and vector field, respectively

$$\int_V \nabla\times(f\mathbf{M})d^3r = \int_S \mathbf{dS}\times(f\mathbf{M})$$

This relation also holds, similar to the divergence theorem but involving the external product. In practice, if the integration volume considered is large enough, so that it includes all the material and has its surface in vacuum where the **M** vector is zero, we have, for a well-behaved **M**(**r**).

$$\int_V \nabla\times(f\mathbf{M})d^3r = 0 \quad \Rightarrow \quad \int \mathbf{M}\times\nabla f\, d^3r = \int f(\nabla\times\mathbf{M})d^3r$$

The contribution to the vector potential due to the magnetisation will thus be:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_V \mathbf{M}(\mathbf{r}')\times\nabla'\frac{1}{|\mathbf{r}-\mathbf{r}'|}d^3r'$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\nabla'\times\mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r'$$

VECTOR POTENTIAL ASSOCIATED WITH THE MAGNETIZATION M(r)

Putting everything together , the total vector potential will be:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\left(\frac{\mathbf{J}_{\text{free}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} + \frac{\nabla\times\mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\right)d^3r'$$

Contribution from free currents — Contribution from magnetization

$\mathbf{B}(\mathbf{r})$ can, at this point, be calculated as $\mathbf{B} = \nabla\times\mathbf{A}$. It is clear, however, that the new integrand term describes the contribution to $A(\mathbf{r})$ of the magnetization current $\mathbf{J}_M$. By inspection of the equation

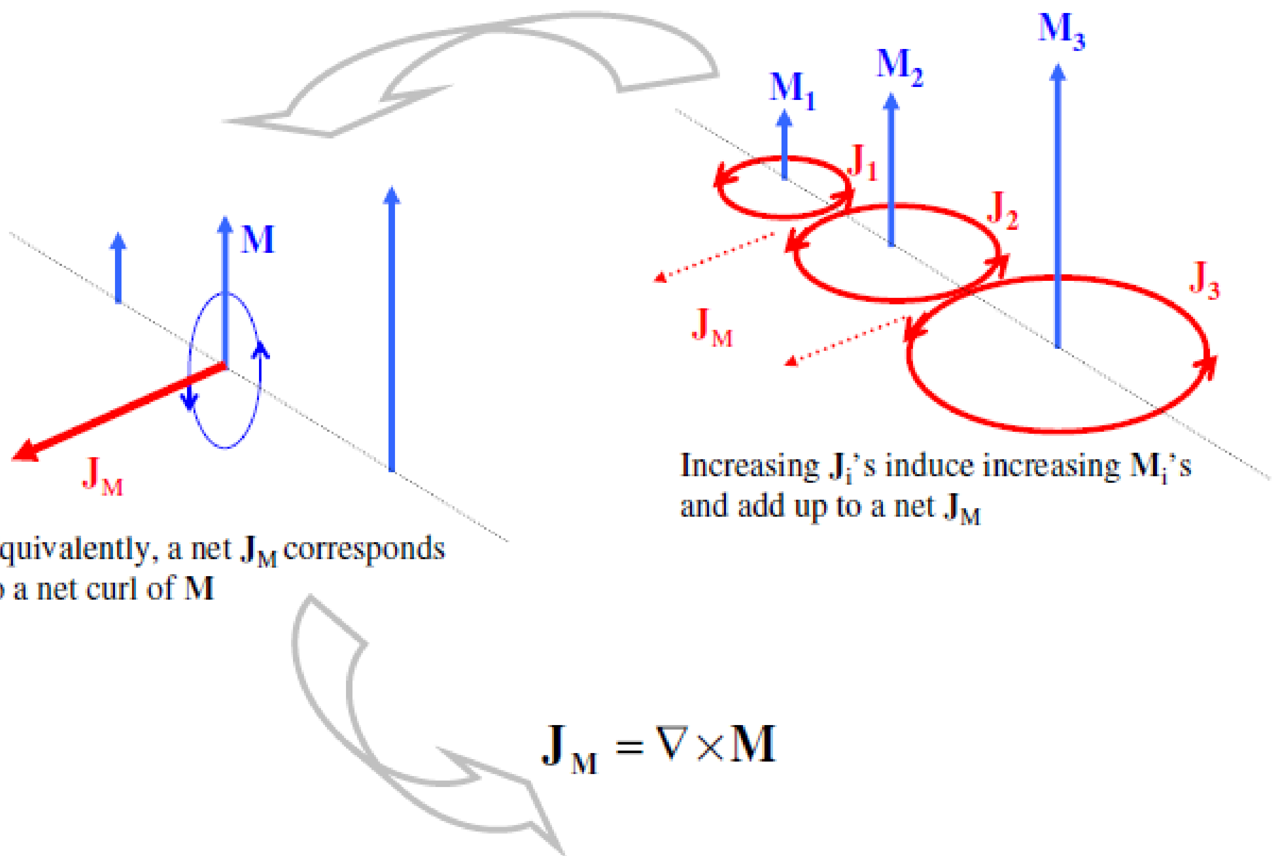
$$\nabla\times\mathbf{B} = \mu_0(\mathbf{J}_{\text{free}} + \mathbf{J}_M)$$

stating that the total magnetic field **B** is generated by the free current $\mathbf{J}_{\text{free}}$ and by the "magnetization" current $\mathbf{J}_M$

$$\mathbf{J}_M = \nabla\times\mathbf{M}$$

→ THE MAGNETIZATION CURRENT DENSITY IS THE CURL OF THE MAGNETIZATION VECTOR

Magnetisation current density $\mathbf{J}_M$ as Curl of $\mathbf{M}$



Equivalently, a net $\mathbf{J}_M$ corresponds to a net curl of $\mathbf{M}$

Increasing $\mathbf{J}_i$'s induce increasing $\mathbf{M}_i$'s and add up to a net $\mathbf{J}_M$

$$\mathbf{J}_M = \nabla \times \mathbf{M}$$

The **H** field

We now look for a field which is generated by the free currents only. This is to some extent analogous to what was done in electrostatics, when we derived the electric displacement **D** which is generated by "free" charges only.

We have:

$$\nabla\times\frac{\mathbf{B}}{\mu_0}=\mathbf{J}_{\text{free}}+\nabla\times\mathbf{M}\quad\Rightarrow\quad\nabla\times\left(\frac{\mathbf{B}}{\mu_0}-\mathbf{M}\right)=\mathbf{J}_{\text{free}}$$

$$\mathbf{H}=\frac{\mathbf{B}}{\mu_0}-\mathbf{M}\quad\rightarrow\quad\nabla\times\mathbf{H}=\mathbf{J}_{\text{free}}$$

THE **H** FIELD IS PRODUCED BY FREE (STEADY) CURRENTS ONLY

We can also write:

$$\mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M})$$

Relation between **H**, **M** and **B**

This reveals the components of **B** due to free currents and to magnetisation, i.e. **H** and **M**. Note that **H** and **M** have the same dimensions (A m^{-1}).

Summary

The equations of MAGNETOSTATICS involving free sources are:

$$\boxed{\nabla \cdot \mathbf{B} = 0} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}), \quad \nabla \times \mathbf{M} = \mathbf{J}_{\mathrm{M}} \quad \Rightarrow \quad \boxed{\nabla \times \mathbf{H} = \mathbf{J}_{\mathrm{free}}}$$

Their counterparts in ELECTROSTATICS are:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \boxed{\nabla \times \mathbf{E} = 0}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \quad \nabla \cdot \mathbf{P} = -\rho_{pol} \quad \Rightarrow \quad \boxed{\nabla \cdot \mathbf{D} = \rho_{\mathrm{free}}}$$

We EMPHASIZE that the fundamental, physical fields are **E** and **B**, although **D** and **H** are useful as they depend on free charges ad currents only, and not on the response of the material. Remember that the two Maxwell equations we have seen so far for these fields are:

$$\boxed{\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \mathbf{B} = 0}$$

MAXWELL EQUATIONS
Also valid the non-static case

Example of Magnetization Calculation

- Exercise 3 (D9.6, Hayt & Buck, 7/e, p. 281):
  - Find: $M = ?$
  - Given:

a) $\mu = 1.8\times10^{-5}$ H/m and $H = 120$ A/m

here we have $M = \chi_m H = (\mu_r - 1)H$

where $\mu_r = \dfrac{\mu}{\mu_0} = \dfrac{1.8\times10^{-5}\text{ H/m}}{4\pi\times10^{-7}\text{ H/m}} = 14.3$

$\therefore \quad M = (13.3)(120\text{ A/m}) = 1600\text{ A/m}$

Example of Magnetization Calculation

- Exercise 3 (continued)
  - Find: $M = ?$
  - Given:

b) $\mu_r = 22$ and $n = 8.3 \times 10^{28}$ atoms/m^3

where $m_i = 4.5 \times 10^{-27}$ A$\cdot$m^2/atom

so here we have $M = n m_i$

$$= (8.3 \times 10^{28})(4.5 \times 10^{-27}) = 374 \text{ A/m}$$

Example of Magnetization Calculation

- Exercise 3 (continued)
  - Find: $M = ?$
  - Given:

c) $B = 300\ \mu\text{T}$ and $\chi_m = 15$

$$\text{where we have } H = \frac{B}{\mu} = \frac{B}{\mu_r \mu_0} = \frac{B}{(1+\chi_m)\mu_0}$$

$$\text{so } M = \chi_m H = \frac{\chi_m B}{(1+\chi_m)\mu_0} = \frac{(15)(300\times 10^{-6}\ \text{T})}{(16)(4\pi\times 10^{-7}\ \text{H/m})}$$

$$\therefore\ M = 224\ \text{A/m}$$